ATME COLLEGE OF ENGINEERING

13th KM Stone, Bannur Road, Mysore - 570 028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

(ACADEMIC YEAR 2023-24)

NOTES

SUBJECT: Transmission and Distribution

SUB CODE: 21EE51

SEMESTER: V

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INSTITUTIONAL VISION AND MISSION

VISION:

- Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

MISSION:

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torchbearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

Department Vision and Mission

Vision:

To create Electrical & Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission:

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

<u>**Program Educational Objectives (PEOs)**</u>

PEO1:

To produce competent and ethical Electrical and Electronics Engineers who will exhibit the necessary technical and managerial skills to perform their duties in society.

PEO2:

To make students continuously acquire and enhance their technical and socio-economic skills.

PEO3:

To aspire students on R&D activities leading to offering solutions and excel in various career paths.

PEO4:

To produce quality engineers who have the capability to work in teams and contribute to real-time projects.

Program Outcomes (POs)

Engineering Graduates will be able to:

PO1: **Engineering Knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization for the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, research literature and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences and engineering sciences.

PO3: Design/Development of Solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health, safety, cultural, societal and environmental considerations.

PO4: Conduct Investigations of Complex Problems: Use research-based knowledge and research methods including design of experiments, analysis, and interpretation of data and synthesis of the information to provide valid conclusions.

PO5: Modern Tool Usage: Create, select and apply appropriate techniques, resources, modern engineering tools and modeling to complex engineering activities with an understanding of the limitations.

PO6: The Engineer and Society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and Sustainability: Understand the impact of the professional engineering solutions in societal, environmental contexts and demonstrate the knowledge of and also the need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics, responsibilities and norms of the engineering practice.

PO9: Individual and Team Work: Function effectively as an individual and as a member or leader in diverse teams and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as: being able to comprehend and write effective reports, design documentation, make effective presentations and receive clear instructions.

PO11: Project Management and Finance: Demonstrate knowledge, understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-Long Learning: Recognize the need for and have the preparation, ability to engage in independent and life-long learning in the broadest context of technological change

Program Specific Outcomes (PSO's)

PSO1: Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics.

PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self-learning.

Transmission and Distribution			
Course Code	**21EE51**	CIE Marks	50
Teaching Hours/Week (L:T:P: S)	2:2:0:0	SEE Marks	50
Total Hours of Pedagogy	40	Total Marks	100
Credits	03	Exam Hours	03

Course objectives:
(1)To understand the concepts of various methods of generation of power.
(2)To understand the importance of HVAC, EHVAC, UHVAC and HVDC transmission.
(3)To design insulators for a given voltage level.
(4)To calculate the parameters of the transmission line for different configurations and assess the performance of the line.
(5)To study underground cables for power transmission and evaluate different types of distribution systems.

Teaching-Learning Process (General Instructions)

These are sample Strategies, which teacher can use to accelerate the attainment of the various course outcomes.

1. Lecturer method (L) needs not to be only traditional lecture method, but alternative effective teaching methods could be adopted to attain the outcomes.
2. Use of Video/Animation to explain functioning of various concepts.
3. Encourage collaborative (Group Learning) Learning in the class.
4. Ask at least three HOT (Higher order Thinking) questions in the class, which promotes critical thinking.
5. Adopt Problem Based Learning (PBL), which fosters students' Analytical skills, develop design thinking skills such as the ability to design, evaluate, generalize, and analyse information rather than simply recall it.
6. Introduce Topics in manifold representations.
7. Show the different ways to solve the same problem with different circuits/logic and encourage the students to come up with their own creative ways to solve them.
8. Discuss how every concept can be applied to the real world - and when that's possible, it helps improve the students' understanding.

Module-1

Introduction to Power System: Structure of electric power system: generation, transmission and distribution. Advantages of higher voltage transmission: HVAC, EHVAC, UHVAC and HVDC. Interconnection. Feeders, distributors and service mains.

Overhead Transmission Lines: A brief introduction to types of supporting structures and line conductors- Conventional conductors; Aluminium Conductor steel reinforced (ACSR), All – aluminium alloy conductor (AAAC) and All –aluminium conductor (AAC). High temperature conductors; Thermal resistant aluminium alloy (ATI),Super thermal resistant aluminium alloy (ZTAI), Gap type thermal resistant aluminium alloy conductor steel reinforced (GTACSR), Gap type super thermal resistant aluminium alloy conductor steel reinforced (GZTACSR). Bundle conductor and its advantages. Importance of sag, Sag calculation – supports at same and different levels, effect of wind and ice. Line vibration and vibration dampers. Overhead line protection against lightening; ground wires.

Overhead Line Insulators: A brief introduction to types of insulators, material used- porcelain, toughened glass and polymer (composite). Potential distribution over a string of suspension insulators. String efficiency, Methods of increasing string efficiency. Arcing horns.

Teaching-Learning Process	Chalk and Board, Power Point Presentation.

Module-2

Line Parameters: Introduction to line parameters- resistance, inductance and capacitance. Calculation of inductance of single phase and three phase lines with equilateral spacing, unsymmetrical spacing, double circuit and transposed lines. Inductance of composite – conductors, geometric mean radius (GMR) and geometric mean distance (GMD). Advantages of single circuit and double circuit lines.). Calculation of capacitance of single phase and three phase lines with equilateral spacing, unsymmetrical spacing, double circuit and transposed lines. Capacitance of composite – conductor, geometric mean radius (GMR) and geometric mean distance (GMD). Advantages of single circuit and double circuit lines.

Teaching-Learning Process	Chalk and Board, Power Point Presentation.

Module-3	
Performance of Transmission Lines: Classification of lines – short, medium and long. Current and voltage relations, line regulation and Ferranti effect in short length lines, medium length lines considering Nominal T and nominal circuits, and long lines considering hyperbolic form equations. Equivalent circuit of a long line. ABCD constants in all cases.	
Teaching-Learning Process	Chalk and Board, Power Point Presentation.
Module-4	
Corona: Phenomena, disruptive and visual critical voltages, corona loss. Advantages and disadvantages of corona. Methods of reducing corona. **Underground Cable:** Types of cables, constructional features, insulation resistance, thermal rating, charging current, grading of cables – capacitance and inter-sheath. Dielectric loss. Comparison between ac and DC cables. Limitations of cables. Specification of power cables.	
Teaching-Learning Process	Chalk and Board, Power Point Presentation.
Module-5	
Distribution: Primary AC distribution systems – Radial feeders, parallel feeders, loop feeders and interconnected network system. Secondary AC distribution systems – Three phase 4 wire system and single phase 2 wire distribution, AC distributors with concentrated loads. Effect of disconnection of neutral in a 3 phase four wire system. **Reliability and Quality of Distribution System:** Introduction, definition of reliability, failure, probability concepts, limitation of distribution systems, power quality, Reliability aids.	
Teaching-Learning Process	Chalk and Board, Power Point Presentation.

Course outcome (Course Skill Set)

At the end of the course the student will be able to :

(1)Explain transmission and distribution scheme, identify the importance of different transmission systems and types of insulators.

(2)Analyze and compute the parameters of the transmission line for different configurations.

(3)Assess the performance of overhead lines.

(4)Interpret corona, explain the use of underground cables.

(5)Classify different types of distribution systems; examine its quality & reliability.

Assessment Details (both CIE and SEE)

The weightage of Continuous Internal Evaluation (CIE) is 50% and for Semester End Exam (SEE) is 50%. The minimum passing mark for the CIE is 40% of the maximum marks (20 marks out of 50). A student shall be deemed to have satisfied the academic requirements and earned the credits allotted to each subject/ course if the student secures not less than 35% (18 Marks out of 50)in the semester-end examination(SEE), and a minimum of 40% (40 marks out of 100) in the sum total of the CIE (Continuous Internal Evaluation) and SEE (Semester End Examination) taken together

Continuous Internal Evaluation:

Three Unit Tests each of **20 Marks (duration 01 hour**)

- First test at the end of 5th week of the semester
- Second test at the end of the 10th week of the semester
- Third test at the end of the 15th week of the semester

Two assignments each of **10 Marks**

- First assignment at the end of 4th week of the semester
- Second assignment at the end of 9th week of the semester

Group discussion/Seminar/quiz any one of three suitably planned to attain the COs and POs for **20 Marks** (**duration 01 hours**)

- At the end of the 13th week of the semester

The sum of three tests, two assignments, and quiz/seminar/group discussion will be out of 100 marks and will be **scaled down to 50 marks**

(to have less stressed CIE, the portion of the syllabus should not be common /repeated for any of the methods of the CIE. Each method of CIE should have a different syllabus portion of the course).

CIE methods /question paper is designed to attain the different levels of Bloom's taxonomy as per the outcome defined for the course.

Semester End Examination:

Theory SEE will be conducted by University as per the scheduled timetable, with common question papers for the subject (**duration 03 hours)**

- The question paper will have ten questions. Each question is set for 20 marks.
- There will be 2 questions from each module. Each of the two questions under a module (with a maximum of 3 sub-questions), **should have a mix of topics** under that module.

The students have to answer 5 full questions, selecting one full question from each module.

Suggested Learning Resources:

Textbooks

1. A Course in Electrical Power, Soni Gupta and Bhatnaghar, DhanpatRai.
2. Principles of Power System, V.K. Mehta, Rohit Mehta S. Chand 1st Edition 2013.

Reference Books

1. Power System Analysis and Design, J. Duncan Gloverat el, Cengage Learning, 4th Edition 2008.
2. Electrical power Generation, Transmission Distribution, S.N. Singh PHI, 2nd Edition, 2009.
3. Electrical Power S.L.Uppal Khanna Publication.
4. Electrical power systems, C. L. Wadhwa, New Age, 5th Edition.
5. Electrical power systems, AshfaqHussain, CBS Publication.
6. Electric Power Distribution, A.S. Pabla, McGraw-Hill, 6th Edition,2012.

Note: For High temperature conductors refer www.jpowers.co.jp/english/product/pdf/gap_c1.pdf and Power.

Activity Based Learning (Suggested Activities in Class)/ Practical Based learning

Activity Based Learning, Quizzes, Seminars.

MODULE 1

MODULE-1

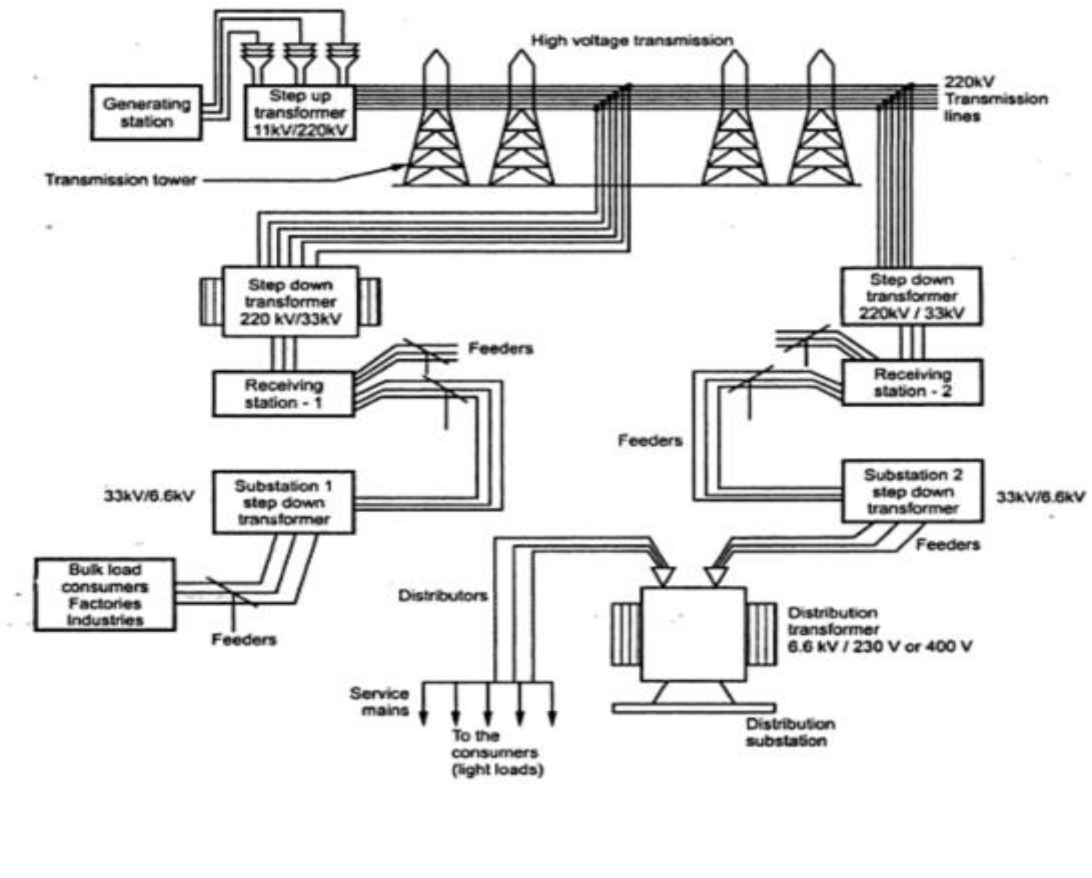
Introduction to power system

A typical Transmission and Distribution Scheme

The flow of electrical power from the generating station to the consumer is called an electrical power system or electrical supply system. It consists of the following important components :

1. Generating station 2. Transmission network 3. Distribution Network

All these important networks are connected with the help of conductors and various step up and step down transformers. A typical transmission and distribution scheme is shown in the Fig. 1



Schematic Representation of a typical transmission distribution scheme

A scheme shows a generating station which is located too far away from cities and towns. It is generating an electrical power at 11 Kv. It is required to increase this level for the transmission purpose.

Hence a step up transformer is used which steps up the voltage level to 220 Kv. This level may be 132 Kv, 220 Kv or more as per the requirement.

Then with the help of transmission lines and the towers, the power is transmitted at very long distances. Design of the transmission lines is based on the factors like transmission voltage level, constants like resistance, reactance of the lines, line performance, interference with the neighbouring circuits etc. Its mechanical features are strength of the supports, sag calculations, tension etc. Transmission of power by the overhead lines is very much cheaper. Similarly the repairs also can be carried out comparatively more easily. The transmission is generally along with additional lines in parallel. These lines are called duplicate lines. Thus two sets of three phase lines work in parallel. This ensures the continuity during maintenance and also can be used to satisfy future demand. The power is then transmitted to the receiving station via step down transformer. This transformer is 220/33 kv or 220/22 Kv transformer.

The power is then transmitted to the substations. A substation consists of a step down transformer of rating 33 KV to 6.6 Kv or 3.3 KV. The transfer of power from receiving station to the substation is with the help of conductors called feeders. This is called secondary transmission.

From the substations, power is distributed to the local distribution centers with the help of distributors. Sometimes for bulk loads like factories and industries, the distributors transfer power directly. For the light loads, there are distribution centers consisting of distribution transformers which step down the voltage level to 230 V or 400 V. This is called primary distribution. In the crowded areas like cities, overhead system of bare conductors is not practicable. In such cases insulated conductors are used on the form of underground cables, to give supply to the consumers. These cables are called service mains. This is called secondary distribution.

This is the complete flow of an electrical power from the generating station to the consumer permises.

Let us study the line diagram of such a typical scheme of transmission and distribution and discuss the various components and voltage levels at the various stages in detail. The Fig. 2 shows the line diagram of a typical transmission and distribution scheme.

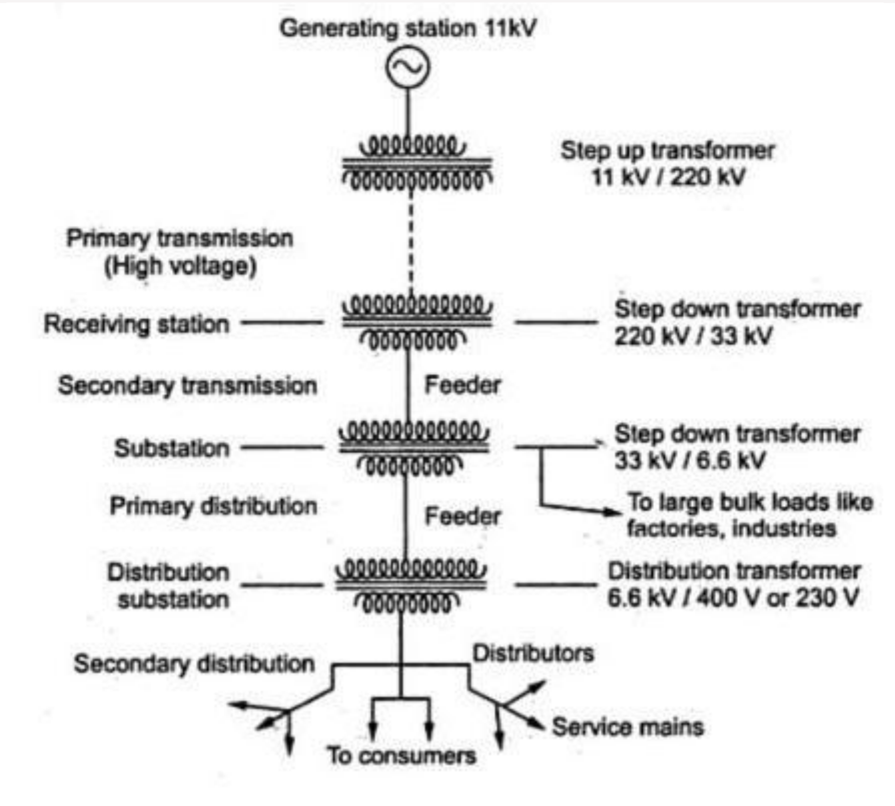


Fig. 2 Line diagram of a typical transmission distribution scheme

At the generating station, an electrical power is generated with the help of three phase alternators running in parallel. In the scheme shown, the voltage level is 11 KV but the voltage level may be 6.6 KV, 22 KV or 33 KV depending upon the capacity of the generating station. After the generating station, actual transmission and distribution starts. The overall scheme can be divided into four sections which are,

1. Primary transmission : It is basically with the help of overhead transmission lines. For the economic aspects, the voltage level is increased to 132 KV, 220 KV or more, with the help of step up transformer. Hence this transmission is also called high voltage transmission. The primary transmission uses 3 phase 3 wire system.

2. Secondary Transmission : The primary transmission line continues via transmission towers till the receiving stations. At the receiving stations, the voltage level is reduced to 22 KV or 33 KV using the step down transformer. There can be more than one receiving stations. Then at reduced voltage level of 22 KV or 33 KV, the power is then transmitted to various substations using overhead 3 phase 3 wire system. This is secondary transmission. The conductors used for the secondary transmission are called feeders.

3. Primary Distribution : At the substation the voltage level is reduced to 6.6 KV, 3.3 KV or 11 KV with the help of step down transformers. It uses three phase three wire underground system. And the power is further transmitted to the local distribution centers. This is primary distribution, also called high voltage distribution. For the large consumers like factories and industries, the power is directly transmitted to such loads from a substation. Such bug loads have their own substations.

4. Secondary Distribution : At the local distribution centers, there are step down distribution transformers. The voltage level of 6.6 KV, 11 KV is further reduced to 400 V using distribution transformers. Sometimes it may be reduced to 230 V. The power is then transmitted using distribution, also called low voltage distribution. This uses 3 phase 4 wire system. The voltage between any two lines is 400 V. while the voltage between any of the three lines and a neutral is 230 V. The single phase lighting loads are supplied using a line and neutral while loads like motors are supplied using three phase lines.

Components of Distribution

The distribution scheme consists of following important components :

1. Substation : Transmission lines bring the power upto the substations at a voltage level of 22 KV or 33 KV. At the substation the level is reduced to 3.3 KV or 6.6 KV. Then using feeders, The power is given to local distribution centers.

2. Local distribution station : It consists of distribution transformer which steps down the voltage level from 3.3 KV, 6.6 KV to 400 V or 230 V. Then it is distributed further using distributors. This is also called distribution substation.

3. Feeders : These are the conductors which are of large current carrying capacitor. The feeders connect the substation to the are where power is to be finally distributed to the consumers. No tapping are taken from the feeders. The feeder current always remains constant.

4. Distributors : These are the conductors used to transfer power from distribution center to the consumers. From the distributors, the tappings are taken for the supply to the consumers.

5. Service Mains : These are the small cables between the distributors and the actual consumers permises.

The interconnection of feeders, distributors and service mains is shown in the Fig. 3.

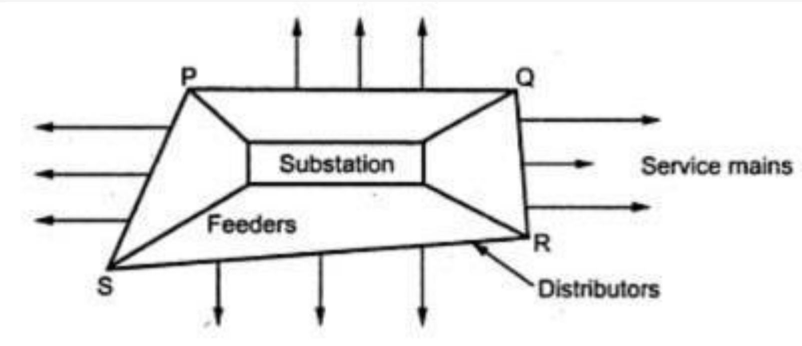


Fig. 3

There is no tapping on feeders. PQ, QR, RS and PS are the distributors which are supplied are supplied by the feeder. The service mains are used to supply the consumers from the distributors. Tappings are taken from the distributors.

n general two types of systems are used for the transmission.

1. Overhead system 2. Underground system

Overhead System

In this system, the transmission of electrical power is by using overhead transmission lines over long distances. In such system, the appropriate spacing is provided between the conductors, at the supports as well as at the intermediate points. This spacing provides insulation which avoids an electric discharge to occur between the conductors. The transmission by overhead system is much cheaper than the underground system. The overhead transmission lines are subjected to the faults occurring due to lightening, short circuits, breakage of line etc. but overhead lines can be easily repaired compared to underground system. It is also true that though such faults are rare, if occurred it is very difficult to find exact point of fault as transmission lines are very long. In the overhead system, the insulation must be provided between the conductor and supporting structure. Hence the maximum stress exists between conductor and earth.

Underground System

The cables are generally preferred in underground system. All the conductors must be insulated from each other in the underground system. As voltage level is high, insulation required is more. Hence due to insulation difficulties, the voltage level used in underground system is below 66 KV while the voltage level used in overhead transmission lines can be as high as 400 KV. The maintenance cost of the underground system is less compared to overhead system. In crowded areas, overhead system using bare conductors is not practicable where underground system using cables is preferred. The line surges are suppressed by using the cables hence cable must be used for the last part of the connection which can save transformers and generators from the damage due to line surges.

In the underground system, the maximum stress exists on the insulation between the conductors.

Looking at the advantages and disadvantages of the two systems, it can be conclude that high voltage transmission is advantageous. Let us steady the effect of increased voltage level of transmission on

1. Volume of copper used for transmission
2. Efficiency of the line
3. The line voltage drop

Effect of High Voltage on Volume of Copper

Let a three phase a.c. system is used for the transmission. The various parameters are,

P = Power transmitted in KW

V = Line voltage in volts

$\cos\Phi$ = Power factor of load

l = Length of line in meters

A = Area of cross-section of conductor in square meters

ρ = Resistivity of conductor material

R = Resistance per conductor in Ω

The resistance per conductor is given by,

$$R = \frac{\rho l}{A} \qquad \dots (1)$$

The load current I can be obtained as,

$$I = \frac{P \times 1000}{\sqrt{3}\, V \cos\phi} \qquad \dots (2)$$

The total copper losses are,

$$W = 3 I^2 R \qquad \dots (3)$$

$$\therefore \quad W = 3 \times \frac{P^2 (1000)^2}{3 V^2 \cos^2\phi} \times \frac{\rho l}{A} \qquad \dots (4)$$

$$\therefore \quad A = \frac{P^2 (1000)^2}{W V^2 \cos^2\phi} \rho l \qquad \dots (5)$$

The volume of copper used is,

Vol = 3 A l

$$\therefore \quad \boxed{Vol = 3 \frac{P^2 (1000)^2}{W V^2 \cos^2\phi} \rho l^2} \qquad \dots (6)$$

It can b seen from the equation (6) that the volume of copper required is inversely proportional to the square of the transmission voltage and the power factor, for given P, W, ρ and l .

Thus greater is the transmission voltage level, lesser is the volume of copper required i.e. the weight of copper used for the conductors. The conductor material required is less, for higher transmission voltage.

Effect of High Voltage on Line Efficiency.

The power input to the line cab be written as, $P_{in} = P_{out} + \text{Losses}$

Now is P_{out} as considered above while the losses are given by the equation (4).

$$\therefore \quad P_{in} = P \times 1000 + \frac{P^2 \times (1000)^2}{V^2 \cos^2\phi} \times \frac{\rho l}{A} \qquad \dots (7)$$

Let J = Current density of conductor in A/m^2

$\therefore$ J = I/A

i.e. $$A = \frac{I}{J} \qquad \ldots(8)$$

$$\therefore \quad P_{in} = P \times 1000 + \frac{P^2 \times (1000)^2}{V^2 \cos^2 \phi} \times \frac{\rho l}{I} \times J \qquad \ldots(9)$$

Using equation (2) in equation (9),

$$P_{in} = P \times 1000 + \frac{P^2 \times (1000)^2}{V^2 \cos^2 \phi} \times \frac{\rho l}{(P \times 1000)} \times \sqrt{3}\, V \cos\phi\, J$$

$$= P \times 1000 + (P \times 1000)\, \frac{\sqrt{3}\rho l J}{V \cos\phi}$$

$$\therefore \quad \boxed{P_{in} = P \times 1000 \left[1 + \frac{\sqrt{3}\rho l J}{V \cos\phi}\right]} \qquad \ldots(10)$$

The line efficiency is given by,

Line efficiency η = Output/Input

$$\therefore \quad \eta = \frac{P \times 1000}{P \times 1000 \left[1 + \frac{\sqrt{3}\rho l J}{V \cos\phi}\right]}$$

$$\therefore \quad \boxed{\eta = \frac{1}{1 + \frac{\sqrt{3}\rho l J}{V \cos\phi}}} \qquad \ldots(11)$$

Mathematically above equation can be approximately written, using Binomial theorem as,

$$\therefore \quad \boxed{\eta = 1 - \frac{\sqrt{3}\rho l J}{V \cos\phi}} \qquad \ldots(12)$$

So for constant values of ρ , l and J, the equation (12) shows that line efficiency is higher for higher transmission voltages.

Effect of High Voltage on Line Drop

Line drop = I x R

Using equation (1),

$$\text{Line drop} = I \times \frac{\rho l}{A} = I \times \frac{\rho l}{\left(\frac{I}{J}\right)} \qquad \text{... Using equation (8)}$$

$$\therefore \quad \text{Line drop} = J \rho l \qquad \text{... (13)}$$

$$\boxed{\%\ \text{Line drop} = \frac{J \rho l}{V} \times 100} \qquad \text{... (14)}$$

The equation (14) shows that higher is the transmission voltage level, lesser is the percentage line drop.

<u>Advantages of High Voltage Transmission</u>

Summarizing the above discussion, the advantages of high voltage transmission can be stated as,

1. The line losses are inversely proportional to the square of voltage and power factor. So line losses are less.
2. For constant losses, the volume of copper required is inversely proportional to the square of the voltage and power factor. Hence the copper required is much less for high voltage transmission.
3. For constant current density, the line efficiency is very for high voltage transmission.
4. The percentage line drop is very small for the high voltage transmission.

It may be noted that along with the voltage level, the power factor also plays an important role. Higher power factor also gives less losses, reduced volume of copper and increased line efficience. Hence consumers are always recommanded to maintain high power factor values.

<u>Disadvantages of High Voltage</u>

Though high voltage transmission offers number of advantages, very high voltage transmission is not practically possible. There is a limit to increase the level of transmission voltage. The high voltage transmission has following limitations.

1. Higher the transmission voltage, higher is the insulation required which can cause problems in connection with conductor supports and clearance between the conductors.
2. Higher insulation means high cost.
3. The cost of transformers, switchgear and other equipments is also high for high voltages.
4. Higher the voltage, sever is the corona effect.

Thus a compromise is necessary to select a transmission voltage. The insulation and other cost must be compensated by reduction in cost due to copper saving.

Practical Transmission and distribution Voltage Levels

Considering the advantages and limitations of high voltage and economical aspects, the following voltage levels are commonly used for the transmission and distribution.

1. For generation : 6.6 KV, 11 KV , 22 KV or 33 KV.
2. For primary transmission : 66 KV, 132 KV, 220 KV upto 400 KV.
3. For secondary transmission : 11 KV , 22 KV or 33 KV
4. For primary distribution : 6.6 KV or 11 KV.
5. For secondary distribution : 230 V and 400V

Types of conductors

When the conductors are used in transmission system for bulk power transfer, then they should fulfil following requirements.

1. They should have low weight.
2. They should have high tensile and fatigue strength.
3. They must have high conductivity.
4. They should have low co-efficient of expansion, low corona loss.
5. They should have less resistance and low cost.

Thus base on conductivity, tensile strength, fatigue strength, corona loss, local conditions and cost, conductors are selected for a particular line. The conductors used in practice are made up from the materials such as copper, aluminium and their alloys.

The advantages of using aluminium conductors over copper conductors are given below.

1. They have low cost.
2. Less resistance and corona loss.
3. Less weight.

But aluminium has less tensile strength, high co-efficient of expansion and large area which restricts its use alone as a conductor.

In order to increase the tensile strength of a conductor, one or more central conductors of different materials are used. These materials give high tensile strength. The different types of aluminium conductors used in power systems with full forms of their abbreviations are as given below.

AAC : - All aluminium conductor.

AAAC : - All aluminium alloy conductor.

ACSR : - Aluminium conductor with steel reinforcment.

ACAR : - Aluminium conductor with alloy reinforcement.

Normally the conductors are stranded as it posses greater flexibility and mechanical strength as compared to single wires of same cross sectional area. In stranded conductors, a central wire is surrounded by successive layers of wires containing 6, 12, 18, 24 ... wires. The consecutive layers are spiralled in opposite directions so as to avoid unwinding. This also makes outer radius of one layer coincide with inner radius of the next.

The stranded conductors are electrically in parallel and spiralled together. Due to use of stranded conductors the skin effect is reduced.

The conductor size is decided based on its current carrying ability and voltage level on which it is working. The total number of conductors in a strand of n layers are given by

Total number of conductors $= 1 + 3n(1 + n) = 3n^2 + 3n + 1$

Overall diameter of stranded conductor with n layer $D = (1 + 2n)\,d$

Here d is diameter of each strand. 7 strand conductor will have one central strand with 6 outer strands each. The size of conductor is specified by its equivalent copper cross sectional area and the number of strands with the diameter of each strand.

Now we will discuss in brief the commonly used conductors.

1.Hard Drawn Copper Conductor

The hard drawn copper conductors are used for overhead lines which provides high tensile strength. These conductors have relatively higher conductivity, long life and high scrap value. The copper conductors are used for distribution network where length of line is short and there are more tappings.

2 Steel Cored Copper Conductor (SCC)

The steel cored copper conductors are made by surrounding a steel core with on or more layers of copper strands. Due to addition of steel core tensile strength of conductor is increased.

3 Cadmium Copper Conductor

With addition of cadmium there is increase in the tensile strength of copper at the cost of decrease in the conductivity. Thus these conductors can be used for longer spans. As tensile strength is increased, longer spans with same sag is possible. The other advantages include easiness in jointing, more resistance to atmospheric corrosion, better resistance to wear and easy machinability. These conductors are carried by smaller supports and are subjected to low wind and ice loadings due to their smaller diameter.

4.Copper Weld Condcutor

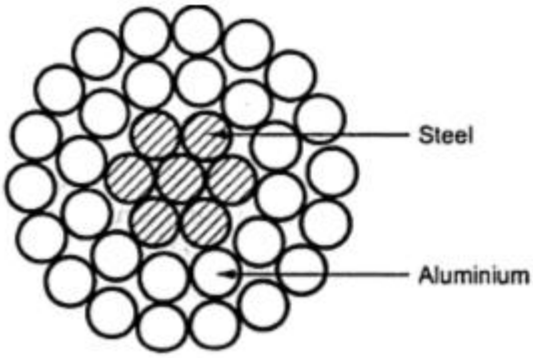
In this type of conductor, copper is welded on to a steel wire by hot rolling and cold drawing a billet of steel coated with copper. The uniform thickness of copper is welded. The conductivity of this conductor lies in the range of 30 to 60% of that of solid copper conductor having same diameter. These are used for longer spans such as river crossings.

5.All Aluminium Conductor (AAC)

Due to increasing cost of copper, aluminium is used in transmission system. Electrolytically refined aluminium is rolled and drawn hard for use as condcutor. For a specific resistance, cross sectional area of aluminium condcutor is greater than that of copper while its weight is about 50% of that of copper conductor. This makes transportation and erection of such conductors economical. Corona effect is reduced due to increased diameter of conductor. These conductors are more used in distribution where transmission lines are short and voltage are lower. There are chances of inter phase faults due to swing if these conductors are employed in the areas where there are high winds. This is because aluminium conductors are lighter, with large conductor area and more sag.

6 Aluminium Conductor with Steel Reinforcement (ACSR)

The mechanical strength that is obtained from conductor made up from all aluminium. This difficulty can be overcome by adding steel core to the conductor. The cross section of this conductor is as shown in the Fig



ACSR conductor

As shown in the Fig. 1 there are 7 steel strands which forms central core. This is surrounded by two layers of around 30 aluminium strands. For a given resistance conductors of different strenghts can be made by taking different properties of steel and aluminium areas. The steel core does not contribute to conduction of current practically. The current carrying capacity and resistance of this conductor is dependent on conductivity of aluminium.

The ACSR conductors are more commonly used as they have following advantages.

1. Due to high mechanical strength and tensile strength, the line span can be increased. The sag is small. So shorter supports are required for line. It is also possible to have longer spans for a given sag. Due to smaller supports, breakdown possibility is low. Insulators and other fittings needed are also less.

2.They have low corona loss.

3. Skin effect is less.

4. These conductors are inexpensive as compared to copper conductors having equal resistance without reduction is efficiency, useful life span and durability.

The disadvantage with ACSR conductor is difficult to make splices and dead ends. There is possibility of corrosion due to electromechanical action between aluminium and steel core. The service conditions decide corrosion rate. This is higher in industrial and coastal areas.

The compacted ACSR conductor or smooth body ACSR conductor is made by pressing conventional ACSR conductor through dies to flatten the aluminium strands into segmental shape. The spaces within the strands are filled while diameter of conductor is reduced. This does not affect electrical and mechanical properties of this conductor. Thus with same aluminium area, diameter of steel core is increased which increases mechanical strength. These conductors can be used for larger span lengths. This is shown in the Fig. 2.

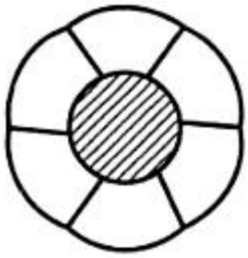


Fig. 2 Compacted ACSR conductor

The expanded conductors are made by adding a plastic or fibrous material between steel core and aluminium strands. This increases diameter of conductor which reduces corona loss and radio interference at extra high voltages. This type of conductor is shown in the Fig.

The filler material such as paper separates the inner steel strands from outer aluminium strands.

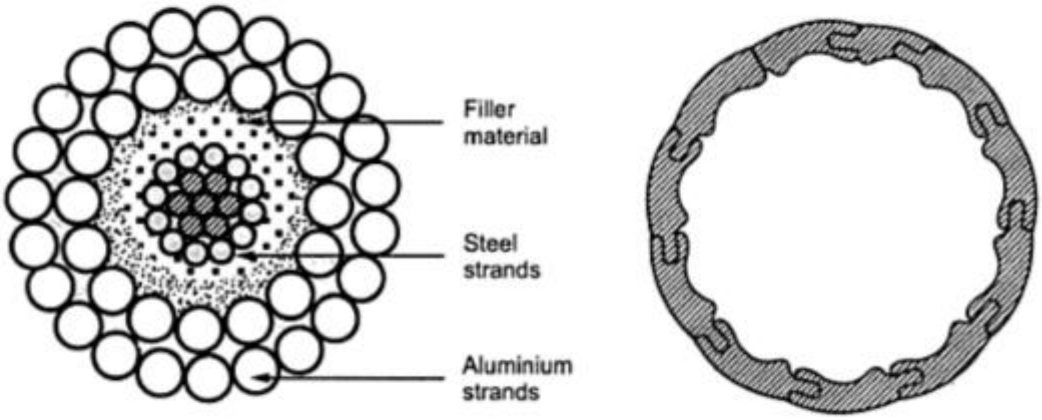


Fig. Expanded conductor

7 All Aluminium Alloy Conductor (AAAC)

The conductor made from aluminium alloys are suitable in urban areas as they provide better tensile strength and condcutivity. These alloys are known with different names in various countries. Some f these alloys are costly as they are heat treated. One of the alloys of aluminium is known as silmalec which contains 0.5% od silicon, 0.5 % of magnesium and rest of aluminium. Due to this there is improvement in conductivity and mechanical strength.

8 ACAR Condcutor

In such conductor, the central core is made up from aluminium alloy which is surrounded by layers of aluminium conductors. The conductivity is better and strength to weight ratio is equal to ACSR conductor having same diameter. As compared to ACSR conductor, ACSR conductor is smaller in size and lower in weight for the same electrical capacity.

9 Phosphor Bronze Conductor

This type of conductor is strong than copper conductor and may be used for longer line spans. The conductivity of such conductor is low which can be improved by use of cadmium-copper core. Phosphor bronze is found to be suitable for atmospheres containing harmful gases.

10 Alumoweld Conductor

In this type of conductor, aluminium is welded on a high strength steel wire. This is costlier as compared to steel cored aluminium (SCA) or ACSR conductor. Around 75% conductor area is covered by aluminium. This is used in earth wires.

11 Galvanized Steel Conductors

This type of conductor is suitable for large length line span or in rural areas where load requirement is comparatively smaller. This type of conductor has high strength. The conductohas large resistance, inductance and voltage drop. The disadvantage with this conductor is it has shorter life.

12 Thermal Resistant Aluminium Alloy conductor(TACSR)

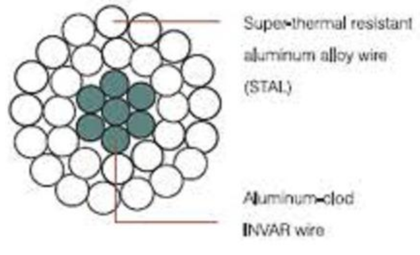
TACSR Conductors are very similar in construction to a conventional ACSR conductor but the EC Grade Aluminum wires are replaced with Hard Drawn Aluminum wires of Heat Resistant Aluminum Alloy (generally known as TAL) . TACSR can be safely operated continuously above 150oC enabling to pump more current through the conductor. Where there is a need to transmit higher power but restrictions on getting new power corridors approved, various Types of TAL conductors are one of the best creative solution options to utilities. Ability of the Zirconium doped aluminum alloy to maintain its electrical and mechanical properties at elevated temperatures makes these conductors a very cost effective solution in refurbishing the existing lines with enhanced capacity.

Features:

- High Current carrying capacity
- Stable at elevated temperatures
- Good mechanical properties
- Economic design
- Best suited for enhancing the existing line capacity where additional power corridors are not feasible.

13. Super Thermal Resistant Aluminium Alloy Conductor Invar Reinforced (STACIR or ZTAI)

Super thermal Alloy is manufactured from aluminium-Zirconium (Al-Zr)alloy rods.The arrangement is shown in figure. The outer layer is made up of Super thermal Resistant aluminium alloy wires.

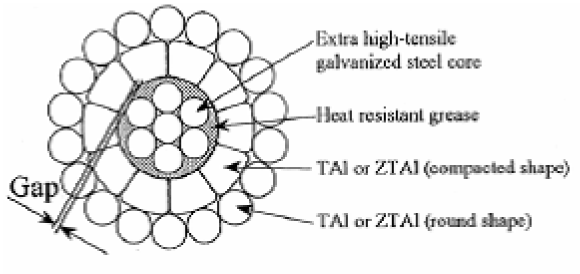


STACIR Conductor

These are concentrically arranged over inner core of aluminium clad INVAR (36% Ni in Steel.The current capacity of this conductor is twice that of light aluminium conductor.The load capacity of the system can be increased simply by replacing existing conductors by STACIR conductors without changing the steel towers. Thus it is very cost effective and stable at higher temperatures.

14. Gap type Thermal Resistant Aluminium Alloy Conductor Steel Reinforced (GTACSR)

It has a unique construction having a small gap between the steel core and super thermal resistant aluminium alloy layer.



The central core is made up of extra high strength steel core.The conductor paer arranged around the core is made up of thermal resistant aluminium alloy.

There is a gap between inner layer of aluminium alloy and the steel core which is filled with grease to avoid the friction.The inner layer of aluminium alloy is trapezoidal in shape to maintain the gap.

The conductor offers excellent sag and current carrying characteristics. It can carry 1.6 times higher current than ACSR conductor of same size. Its cost is low and construction period is short.To increase the existing capacity, the existing conductors can be simply replaced by GTACSR conductors without changing the towers. This construction allows low sag properties and good mechanical strength.

15. Gap type Super Thermal Resistant Aluminium Alloy Conductor Steel Reinforced (GZTACSR)

The construction is similar to GTACSR but the outer aluminium conductors are made up of heat resistant zirconium aluminium alloy. This makes the conductor well suited for the continous operation at elevated temperature upto 210^0C, without affecting its mechanical & electrical properties. Thus this conductor provides very operation at high temperatures.

The various features are

1.Can carry two times higher current than ACSR conductor of same size.

2.Suitable for the continous operation at high temperatures.

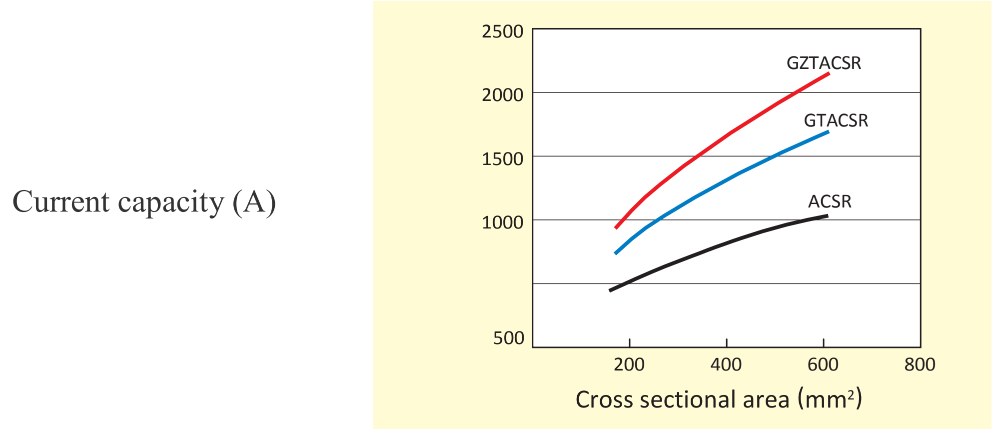
3.Low sag at high temperatures.

4.Very good mechanical and electrical properties at high temperatures.

5.Low thermal knee point.

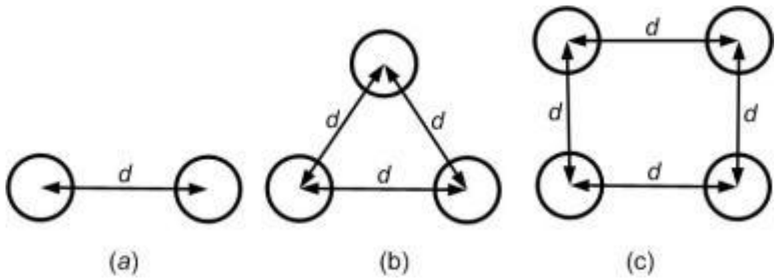
6.Economical for increasing the overall capacity of lines, simply by replacing existing lines without changing the towers.

Fig.shows the comparison of current capacities of ACSR, GTACSR AND GZTASCR conductors of various sizes.



16.Bundled Conductors

For high voltage transmission more than 220KV,two or more conductors are used per phase in close proximity but not touching each other.Such conductors are called bundled conductors.All such conductors belonging to one phase are grouped are grouped together by a metallic structure called spacers.The spacers are used to maintain constant distance between the conductors throughout the length, avoiding touching of conductors amongst themselves.



Each conductor joined by the spacer belongs to the same phase.There are three such groups of conductors in single circuit as shown in the fig. In double circuit transmission , there are six groups of conductors.

Advantages

1.Reduced reactance of the line due to increase in self geometric mean distance.

2. The maximum power transfer capability of the line increases.

3. There is increase in the surge impedance loading.

4.Increase in the capacity of the line.

5. Reduces the effect of corona and radio interference.

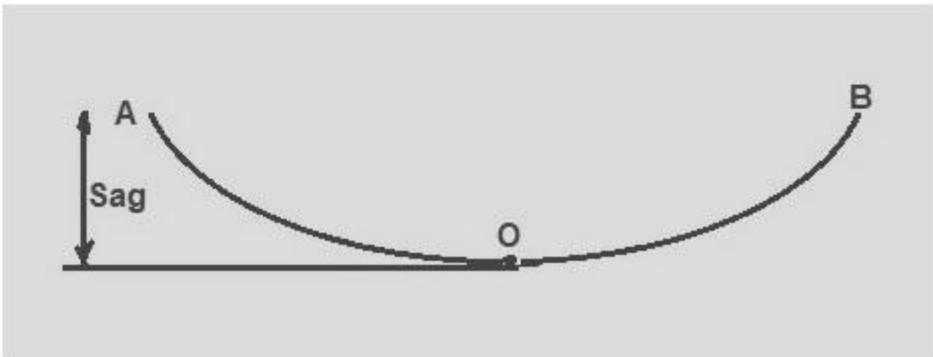
6. reduces voltage gradient in conductor surface.

7.The current carrying capacity of the line increases due to bundled conductors.
8.It increases effective surface area exposed to air hence it has better and efficient cooloing.
9.It has reduced influence of skin effect.
The only limitation is that bundled conductors experience greater wind loading than single conductors.

OVERHEAD TRANSMISSION LINES-

Sag in Overhead Transmission Line and Its Calculation

Sag in overhead Transmission line conductor refers to the difference in level between the point of support and the lowest point on the conductor.



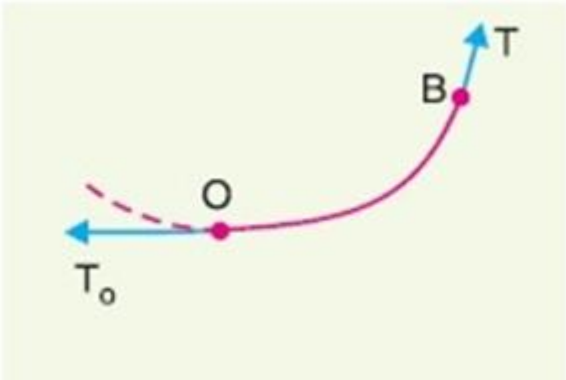
As shown in the figure above, a Transmission line is supported at two points A and B of two different Transmission Towers. It is assumed that points A and B are at the same level from the ground. Therefore as per our definition of Sag, difference in level of point A or B and lowest point O represents the Sag

Sag in Transmission line is very important. While erecting an overhead Transmission Line, it should be taken care that conductors are under safe tension. If the conductors are too much stretched between two points of different Towers to save conductor material, then it may happen so that the tension is conductor reaches unsafe value which will result conductor to break.

Therefore, in order to have safe tension in the conductor, they are not fully stretched rather a sufficient dip or Sag is provided. The dip or Sag in Transmission line is so provided to maintain tension in the conductor within the safe value in case of variation in tension in the conductor because of seasonal variation. Some very basic but important aspects regarding Sag are as follows:

1) As shown in the figure above, if the point of support of conductor is at same level from the ground, the shape of Sag is Catenary. Now we consider a case where the point of support of conductor are at same level

but the Sag is very less when compared with the span of conductor. Here span means the horizontal distance between the points of support. In such case, the Sag-span curve is parabolic in nature.



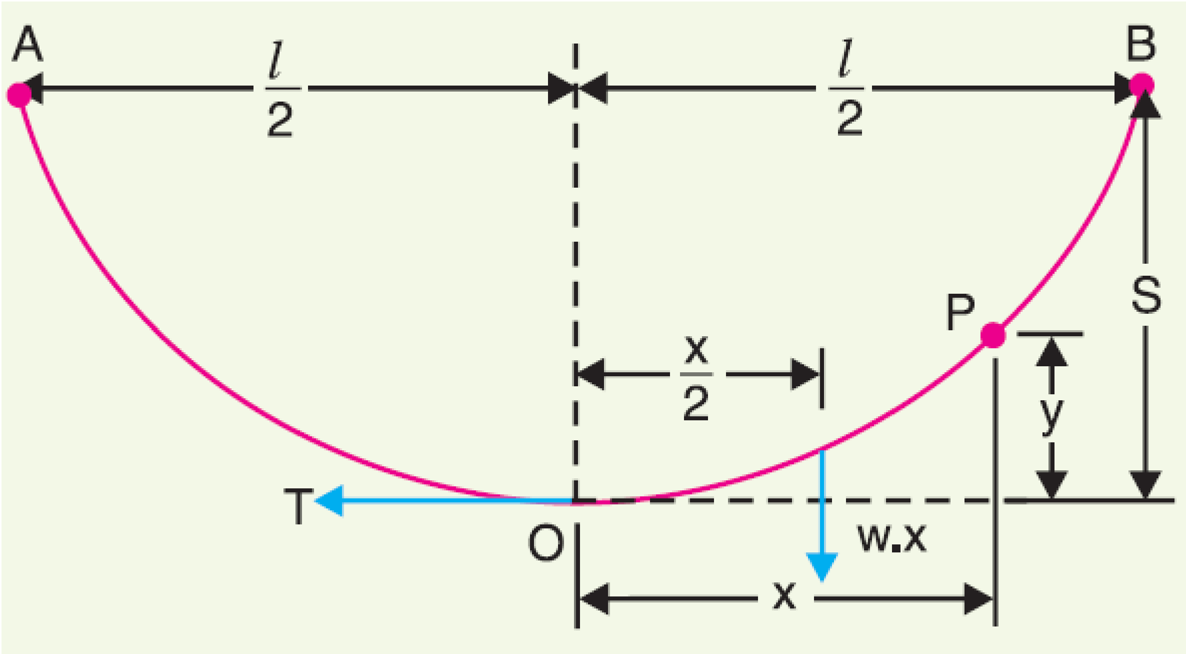
2) The tension at any point on the conductor acts tangentially as shown in figure above. Thus the tension at the lowest point of the conductor acts horizontally while at any other point we need to resolve the tangential tension into vertical and horizontal component for analysis purpose. The horizontal component of tension remains constant throughout the span of conductor.

Calculation of Sag:

As discussed earlier in this post, enough Sag shall be provided in overhead transmission line to keep the tension within the safe limit. The tension is generally decided by many factors like wind speed, ice loading, temperature variations etc. Normally the tension in conductor is kept one half of the ultimate tensile strength of the conductor and therefore safety factor for the conductor is 2.

Case1: When the conductor supports are at equal level.

Let us consider an overhead line supported at two different towers which are at same level from ground. The point of support are A and B as shown in figure below. O in the figure shows the lowest point on the conductor. This lowest point O lies in between the two towers i.e. point O bisects the span equally.



Let,

L = Horizontal distance between the towers i.e. Span

W = Weight per unit length of conductor

T = Tension in the conductor

Let us take any point P on the conductor. Assuming O as origin, the coordinate of point P will be (x,y).

Therefore, weight of section OP = Wx acting at distance of x/2 from origin O.

As this section OP is in equilibrium, hence net torque w.r.t point P shall be zero.

Torque due to Tension T = Torque due to weight Wx

Ty = Wx(x/2)

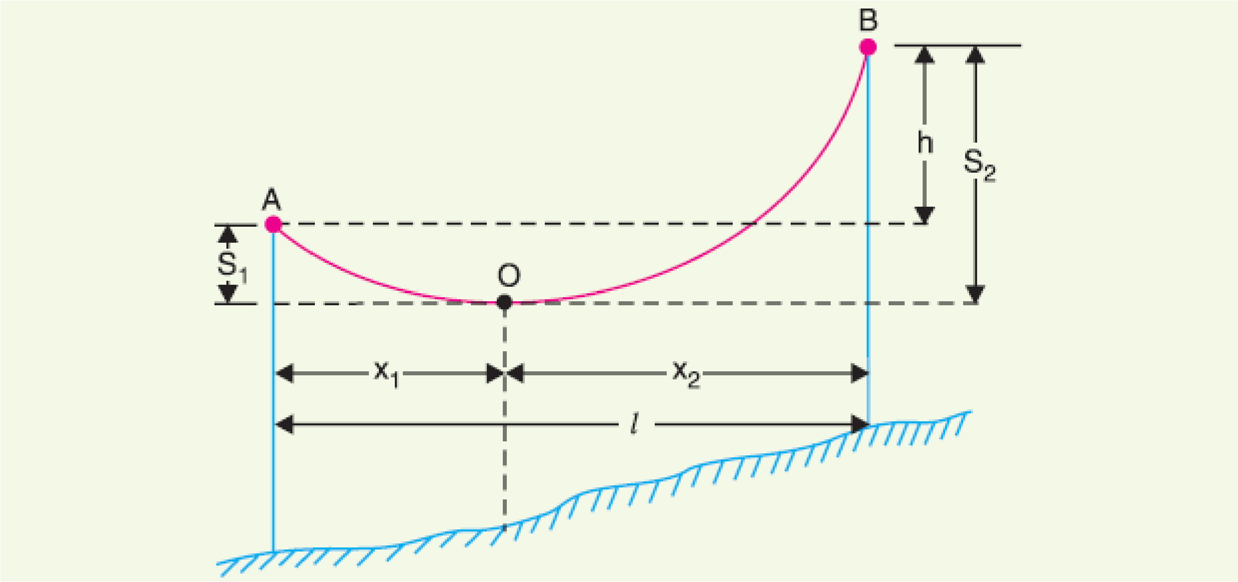
Therefore, **$y = Wx^2 / 2T$ ……………………….(1)**

For getting Sag, put x = L/2 in equation (1)

Sag = $WL^2/8T$

Case2: When the conductor supports are at unequal level.

In hilly area, the supports for overhead transmission line conductor do not remain at the same level. Figure below shows a conductor supported between two points A and B which are at different level. The lowest point on the conductor is O.



Let,

L = Horizontal distance between the towers i.e. Span

H = Difference in level between the two supports

T = Tension in the conductor

X_1 = Horizontal distance of point O from support A

X_2 = Horizontal distance of point O from support B

W = Weight per unit length of conductor

From equation (1),

Sag $S_1 = WX_1^2/2T$

and Sag $S_2 = WX_2^2/2T$

Now,

$S_1 – S_2 = (W/2T)[X_1^2 – X_2^2]$

$= (W/2T)(X_1 – X_2)(X_1 + X_2)$

But $X_1 + X_2 = L$ ……………………..(2)

So,

$S_1 – S_2 = (WL/2T)(X_1 – X_2)$

$X_1 – X_2 = 2(S_1 – S_2)T / WL$

$X_1 – X_2 = 2HT / WL$ (As $S_1 – S_2 = H$)

$X_1 – X_2 = 2HT / WL$ ………………..(3)

Solving equation (2) and (3) we get,

$X_1 = L/2 – TH/WL$

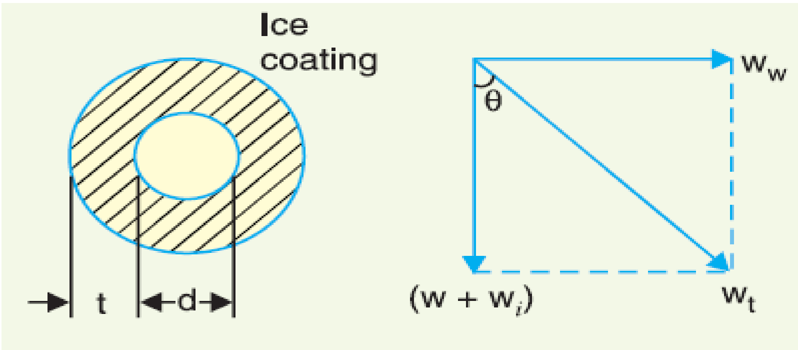
$X_2 = L/2 + TH/WL$

By putting the value of X_1 and X_2 in Sag equation, we can easily find the value of $S_1$and S_2.

The above equations for Sag are only valid in ideal situation. Ideal situation refers to a condition when no wind is flowing and there is no any effect of ice loading. But in actual practise, there always exists a wind pressure on the conductor and as far as the ice loading is concerned, it is mostly observed in cold countries. In a country like India, ice loading on transmission line is rarely observed.

Effect of Wind and Ice Loading on Sag:

Coating of ice on conductor (it is assumed that ice coating is uniformly distributed on the surface of conductor) increases the weigh of the conductor which acts in vertically downward direction. But the wind exerts a pressure on the conductor surface which is considered horizontal for the sake of calculation.



As shown in figure above, net weight acting vertically downward is sum of weight of ice and weight of conductor.

Therefore,

Net weight of conductor per unit length $W_t = \sqrt[2]{(Wi + W)^2 + Ww^2}$

Here,

W = Weight of conductor per unit length

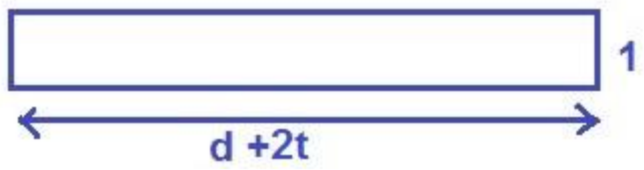
Wi = Weight of ice per unit length

Ww = Wind force per unit length

= Wind PressurexArea

= Wind Presuurex(2d+t)x1

Note the way of calculation of Area of conductor. What I did, I just stretched the conductor along the diameter to make a rectangle as shown in figure below.



Thus from equation (1),

Sag = $W_t L^2/2T$

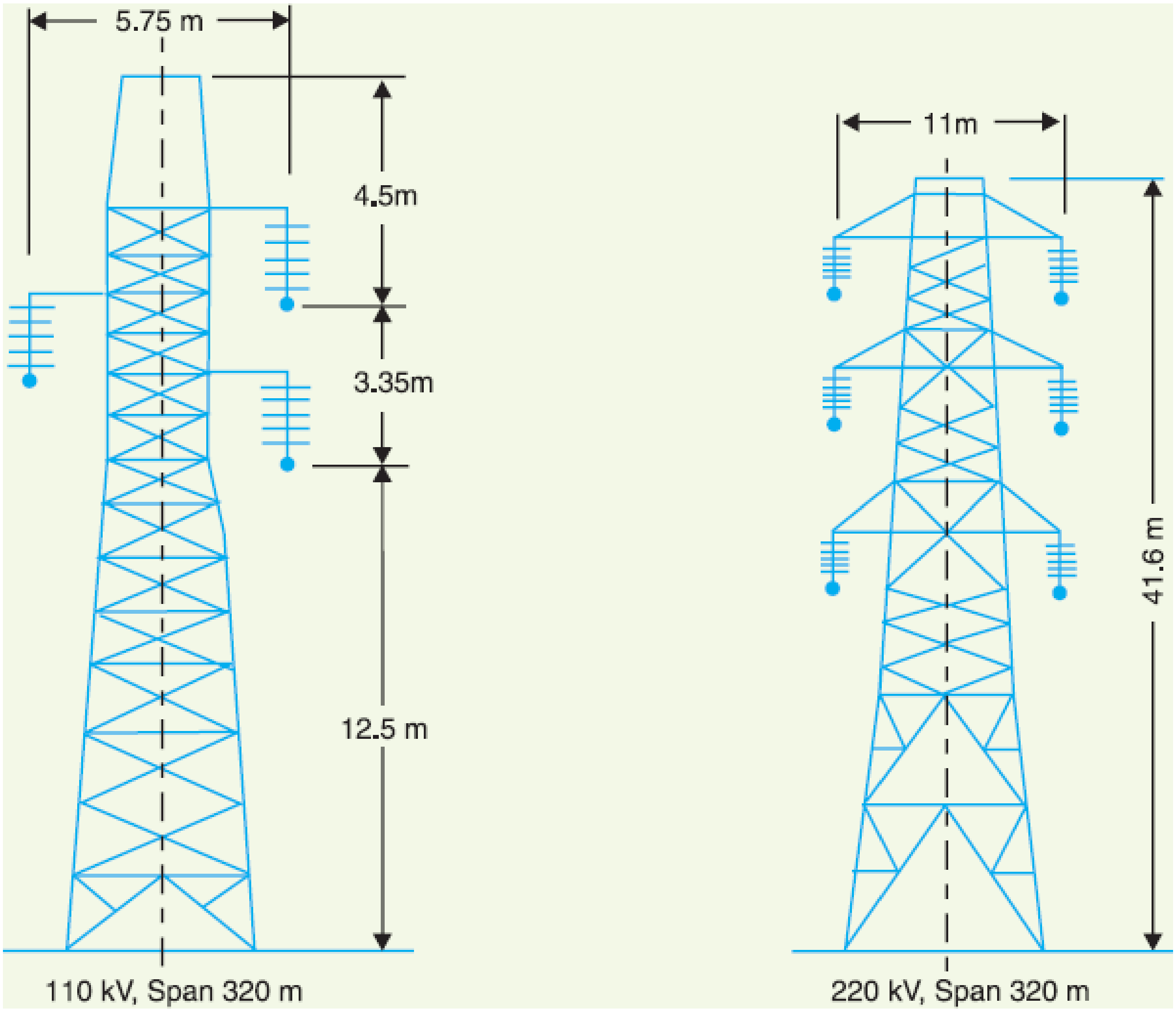
And the angle made by conductor from vertical = tanϴ

$= W_w / (W+W_i)$

INSULATORS

The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports i.e., line conductors must be properly insulated from supports. This is achieved by securing line conductors to supports with the help of insulators. The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth. In general, the insulators should have the following desirable properties :

High mechanical strength in order to withstand conductor load, wind load etc.



High electrical resistance of insulator material in order to avoid leakage currents to earth.

High relative permittivity of insulator material in order that dielectric strength is high.

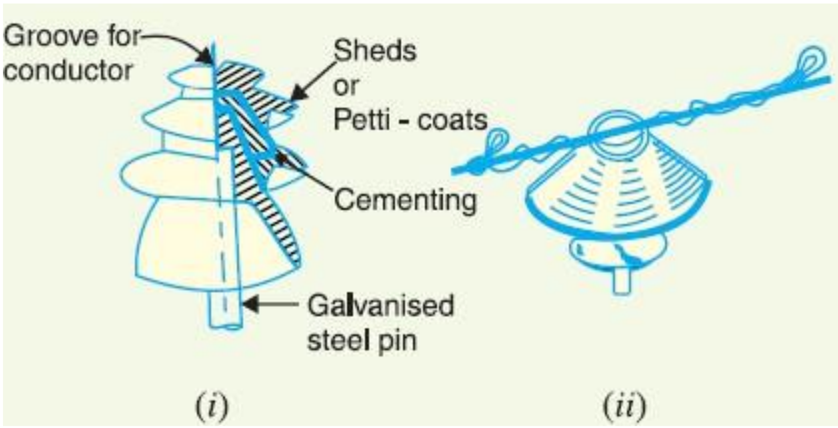
The insulator material should be non-porous, free from impurities and cracks otherwise the permittivity will be lowered.

. It is stronger mechanically than glass, gives less trouble from leakage and is less effected by changes of temperature.

Types of Insulators

The successful operation of an overhead line depends to a considerable extent upon the proper selection of insulators. There are several types of insulators but the most commonly used are pin type, suspension type, strain insulator and shackle insulator.

1. **Pin type insulators**. The part section of a pin type insulator is shown in Fig. As the name suggests, the pin type insulator is secured to the cross-arm on the pole. There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor Pin type insulators are used for transmission and distribution of electric power at voltages upto 33 kV. Beyond operating voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.



Causes of insulator failure. Insulators are required to withstand both mechanical and electrical stresses. The latter type is pirmarily due to line voltage and may cause the breakdown of the insulator. The electrical breakdown of the insulator can occur either by flash-over or puncture. In flashover, an arc occurs between the line conductor and insulator pin (i.e., earth) and the discharge jumps across the *air gaps, following shortest distance.

Fig. shows the arcing distance (i.e. a + b + c) for the insulator. In case of flash-over, the insulator will continue to act in its proper capacity unless extreme heat produced by the arc destroys the insulator. In case of puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown is involved, the insulator is permanently destroyed due to excessive heat. In practice, sufficient thickness of porcelain is provided in the insulator to avoid puncture by the line voltage.

It is desirable that the value of safety factor is high so that flash-over takes place before the insulator gets punctured. For pin type insulators, the value of safety factor is about 10.

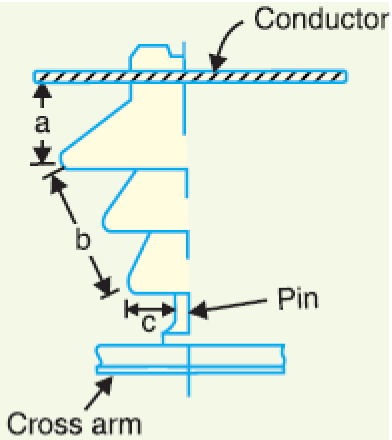
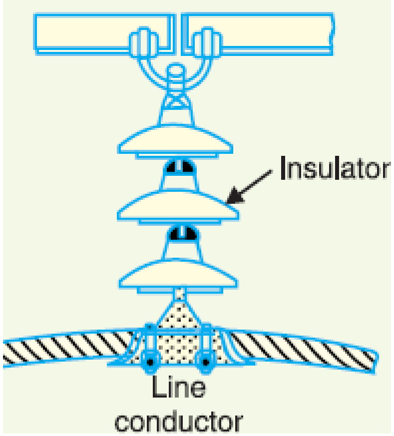


Fig. 8.6



2 **Suspension type insulators**. The cost of pin type insulator increases rapidly as the working voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV. For high voltages (>33 kV), it is a usual practice to use suspension type insulators shown in Fig. . They consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

Advantages

Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.

Each unit or disc of suspension type insulator is designed for low voltage,usually 11 kV. Depending upon the working voltage, the desired number of discs can be connected in series.

If any one disc is damaged, the whole string does not become useless because the damaged disc can be replaced by the sound one.

The suspension arrangement provides greater flexibility to the line. The connectio cross arm is such that insulator string is free to swing in any direction and can take up the position where mechanical stresses are minimum.

In case of increased demand on the transmission line, it is found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. The additional insulation required for the raised voltage can be easily obtained in the suspension
arrangement by adding the desired number of discs.

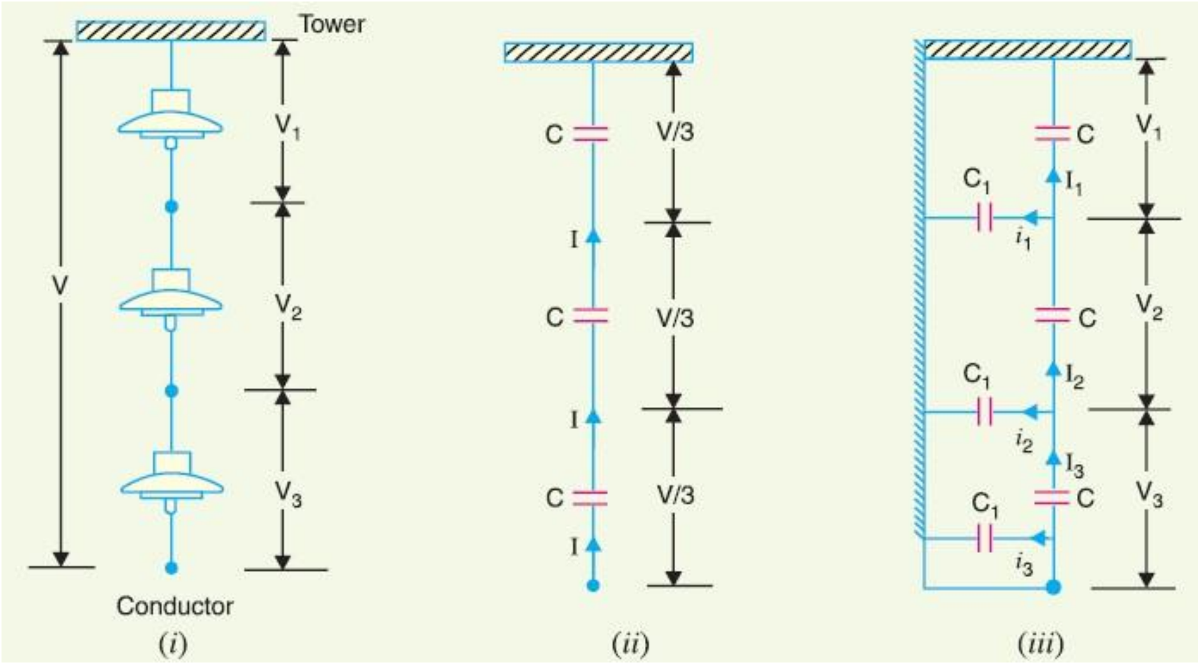
The suspension type insulators are generally used with steel towers. As the conductors run below the earthed cross-arm of the tower, therefore, this arrangement provides partial protection from lightning.

Strain insulators. When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used. For low voltage lines (< 11 kV), shackle insulators are used as strain insulators. However,
for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in Fig. The discs of strain insulators are used in the vertical plane.

3.Shackle insulators. In early days, the shackle insulators were used as strain insulators. But now a days, they are frequently used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly fixed to the pole with a bolt or to the cross arm. Fig. shows a shackle insulator fixed to the pole. The conductor in the groove is fixed with a soft binding wire.

Potential Distribution over Suspension Insulator String

A string of suspension insulators consists of a number of porcelain discs connected in series through metallic links. Fig. (i) shows 3-disc string of suspension insulators. The porcelain portion of each disc is inbetween two metal links. Therefore, each disc forms a capacitor C as shown in Fig. (ii). This is known as mutual capacitance or self-capacitance. If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same i.e., V/3 as shown in Fig. (ii). However, in actual practice, capacitance also exists between metal fitting of each disc and tower or earth. This is known as shunt capacitance C_1. Due to shunt capacitance, charging current is not the same through all the discs of the string [See Fig. (iii)]. Therefore, voltage across each disc will be different. Obviously, the disc nearest to the line conductor will have the maximum* voltage. Thus referring to Fig.(iii), V_3 will be much more than V_2 or V_1.



The following points may be noted regarding the potential distribution over a string of suspension insulators :

The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs due to the presence of shunt capacitance.

The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc goes on decreasing.

The unit nearest to the conductor is under maximum electrical stress and is likely to be punctured. Therefore, means must be provided to equalise the potential across each unit. This is fully discussed in Art.

If the voltage impressed across the string were d.c., then voltage across each unit would be the same. It is because insulator capacitances are ineffective for d.c.

String Efficiency

As stated above, the voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and is usually expressed in terms of string efficiency.

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as string efficiency i.e.,

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

where n = number of discs in the string.

Mathematical expression. Fig. 8.11 shows the equivalent circuit for a 3-disc string. Let us suppose that self capacitance of each disc is C. Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance *i.e.*, $C_1 = KC$. Starting from the cross-arm or tower, the voltage across each unit is V_1, V_2 and V_3 respectively as shown.

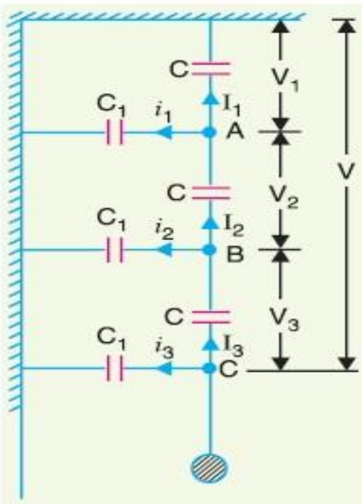


Fig. 8.11

Applying Kirchhoff's current law to node A, we get,

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C^* = V_1 \omega C + V_1 \omega C_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 \omega K C$$

$$\therefore \quad V_2 = V_1 (1 + K) \quad \ldots(i)$$

Applying Kirchhoff's current law to node B, we get,

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega C_1 \dagger$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega K C$$

or $$V_3 = V_2 + (V_1 + V_2) K$$

$$= KV_1 + V_2 (1 + K)$$

$$= KV_1 + V_1 (1 + K)^2 \quad [\because V_2 = V_1 (1 + K)]$$

$$= V_1 [K + (1 + K)^2]$$

$$\therefore \quad V_3 = V_1 [1 + 3K + K^2] \quad \ldots(ii)$$

Voltage between conductor and earth (*i.e.*, tower) is

$$V = V_1 + V_2 + V_3$$

$$= V_1 + V_1(1 + K) + V_1 (1 + 3K + K^2)$$

$$= V_1 (3 + 4K + K^2)$$

$$\therefore \quad V = V_1 (1 + K)(3 + K) \quad \ldots(iii)$$

From expressions (*i*), (*ii*) and (*iii*), we get,

$$\frac{V_1}{1} = \frac{V_2}{1 + K} = \frac{V_3}{1 + 3K + K^2} = \frac{V}{(1 + K)(3 + K)} \quad \ldots(iv)$$

$\therefore$ Voltage across top unit, $$V_1 = \frac{V}{(1 + K)(3 + K)}$$

String efficiency is an important consideration since it decides the potential distribution along the string. The greater the string efficiency, the more uniform is the voltage distribution. Thus 100%
string efficiency is an ideal case for which the volatge across each disc will be exactly the same. Although it is impossible to achieve 100% string efficiency, yet efforts should be made to improve it as close to this value as possible.

$$\text{Voltage across second unit from top, } V_2 = V_1(1 + K)$$

$$\text{Voltage across third unit from top, } V_3 = V_1(1 + 3K + K^2)$$

$$\text{\%age String efficiency} = \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to conductor}} \times 100$$

$$= \frac{V}{3 \times V_3} \times 100$$

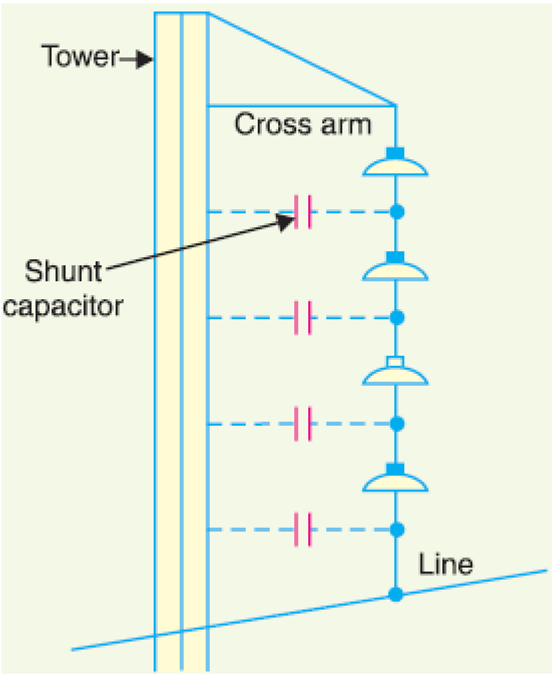
The following points may be noted from the above mathematical analysis :
If K = 0·2 (Say), then from exp. (iv), we get, V2 = 1·2 V1 and V3 = 1·64 V1. This clearly shows that disc nearest to the conductor has maximum voltage across it; the voltage across other discs decreasing progressively as the cross-arm in approached.
The greater the value of K (= C1/C), the more non-uniform is the potential across the discs and lesser is the string efficiency.
The inequality in voltage distribution increases with the increase of number of discs in the string. Therefore, shorter string has more efficiency than the larger one.

Methods of Improving String Efficiency

It has been seen above that potential distribution in a string of suspension insulators is not uniform. The maximum voltage appears across the insulator nearest to the line conductor and decreases progressively as the crossarm is approached. If the insulation of the highest stressed insulator (i.e. nearest to conductor) breaks down or flash over takes place, the breakdown of other units will take place in succession. This necessitates to equalise the potential across the various units of the string i.e. to improve the string efficiency. The various methods for this purpose are :

By using longer cross-arms.

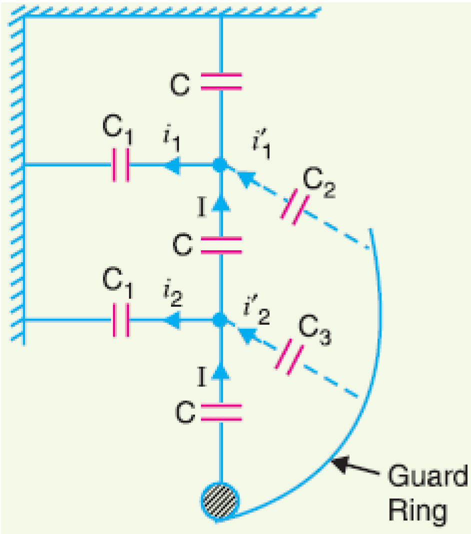
The value of string efficiency depends upon the value of K i.e., ratio of shunt capacitance to mutual capacitance. The lesser the value of K, the greater is the string efficiency and more uniform is the voltage distribution. The value of K can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased i.e., longer cross-arms should be used. However, limitations of cost and strength of tower do not allow the use of very long cross-arms. In practice, $K = 0{\cdot}1$ is the limit that can be achieved by this method.



By grading the insulators. In this method, insulators of different dimensions are so chosen that each has a different capacitance. The insulators are capacitance graded i.e. they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit (i.e., nearest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalise the potential distribution across the units in the string. This method has the disadvantage that a large number of different-sized insulators are required. However, good results can be obtained by using standard insulators for most of the string and larger units for that near to the line conductor.

By using a guard ring.

The potential across each unit in a string can be equalised by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in the Fig. The guard ring introduces capacitance between metal fittings and the line conductor. The guard ring is contoured in such a way that shunt capacitance currents i1, i2 etc. are equal to metal fitting line capacitance currents i□1, i□2 etc. The result is that same charging current I flows through each unit of string. Consequently, there will be uniform potential distribution across the units.



Important Points

While solving problems relating to string efficiency, the following points must be kept in mind:

The maximum voltage appears across the disc nearest to the conductror (i.e., line conductor).

The voltage across the string is equal to phase voltage i.e., Voltage across string =

Voltage between line and earth = Phase Voltage

Line Voltage = 3 √ Voltage across string

Question Bank

1. Draw the line diagram of a typical power supply scheme indicating the standard voltages.
2. Write short note on advantages of HV transmission.
3. Write short note on H V D C transmission.
4. Write short note on feeders, distributors and service mains.
5. Write the comparison betn. Overhead and underground transmission system.
6. What are the advantages and limitations of high voltage a.c transmission?

Overhead Transmission Lines

1. Show that a transmission line conductor suspended between level support assumes the shape of a catenory. Derive the expression for sag.
2. Explain what is sag and why it is inevitable in over head transmission lines? What are the factors influencing it?
3. With usual notations derive an expression for maximum sag of a tranmn. Line where the supports are at different levels?
4. Obtain the expression for sag in a freely suspended conductor when the supports are at equal levels.
5. Explain the effects of sag in overhead trasmn. Line.
6. Obtain the expression for sag in a power conductor when the supports are at equal levels, taking into the effect of wind and ice loading
7. Write short note on effect of ice load and wind effect on sag of transmn. Line.
8. From the first principles derive the expression for sag in a freely suspended conductor when the supports are at unequal levels

Insulators

1 Explain the various tests conducted on insulators.

With usual notations, derive the general expression for the metal link of string to line capacitance, when guard ring is used for the string of insulators.

3. What is string efficiency in the context of suspension insulators? Explain the methods of improving the same.
4. Write short note on different types of O H line insulators.
5. Define string efficiency and hence calculate the mathematical expression for it.
6. Explain the methods of improving the string efficiency.
7. What are the insulators with O H Lines? Discuss the desirable properties of insulators and name the types of insulators.

MODULE 2

MODULE-2

Line Parameters.

The electric power generated in the generating station is transmitted with the help of transmission line which are normally overhead. The parameters associated with these transmission lines are inductance, capacitance, resistance and conductance. The performance of the transmission line is dependent on all these parameters. These parameters are uniformly distributed along the length of transmission line. The efficiency and voltage regulation whether it is good or poor is determined from these constants. For good electric design of transmission line, a sound knowledge about all these parameters is essential.

Conductance between conductors or between conductors and the ground accounts for the leakage current at the insulators of overhead lines and through the insulation of cables. The leakage at insulators of over head lines is negligible, the conductance of an overhead line is assumed to be zero.

Transmission Line Constants

Transmission lines are the circuits with the distributed constants such as resistance, inductance, conductance and capacitance distributed along the length of transmission line. These constant concentrated at any point. Their values are normally expressed as per kilometer length of line.

Consider the distributed circuit shown in the Fig. 1. The analysis of this circuit can be made by lumping the parameters which is shown in the Fig. 2.

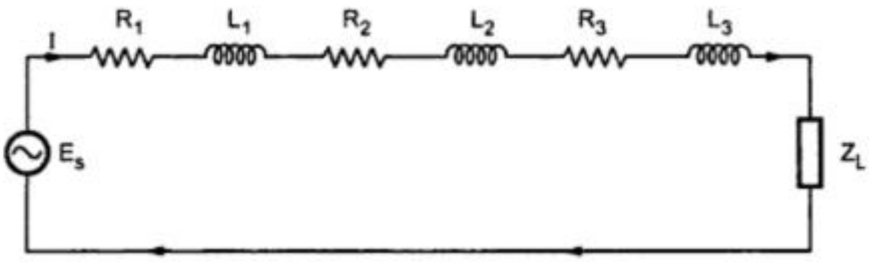


Fig. 1

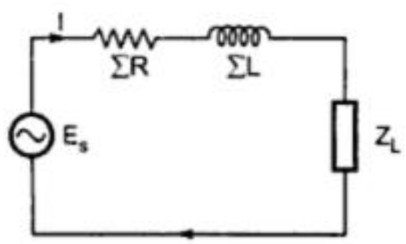


Fig. 2

The analysis of circuit becomes easier by lumping the parameters as the same current is passing through each element of resistance and inductance.

Similarly the circuits shown in the Fig. 3(a) and (b) are seen to be equivalent as the same voltage is applied across each conductance and capacitance.

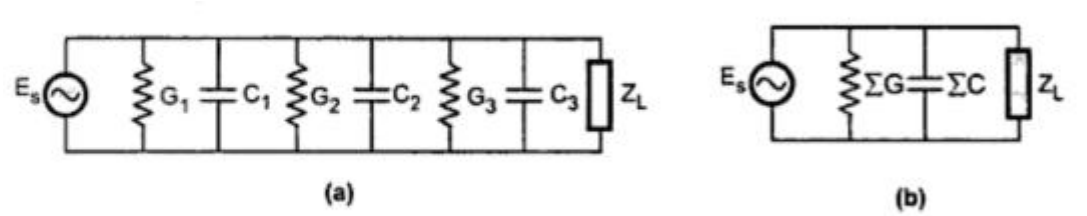
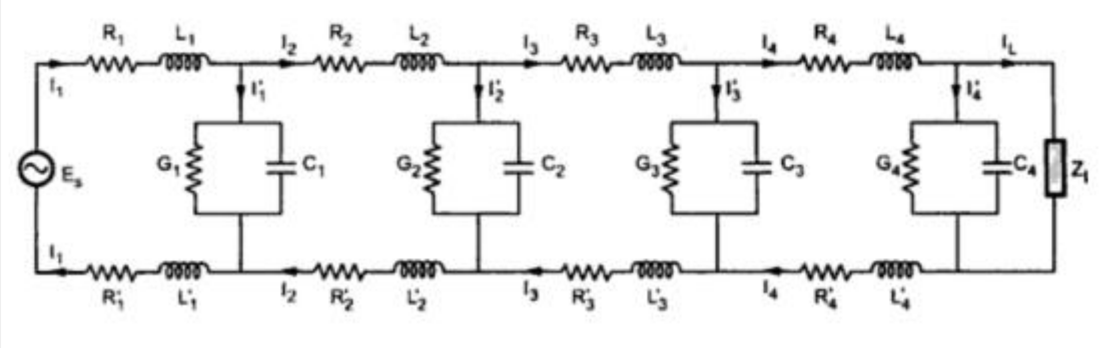


Fig. 3

But if the elements are distributed as shown in the Fig. 4. then the elements can not be lumped as the same current is not flowing through series element and the voltage across different element changes from one section to other.

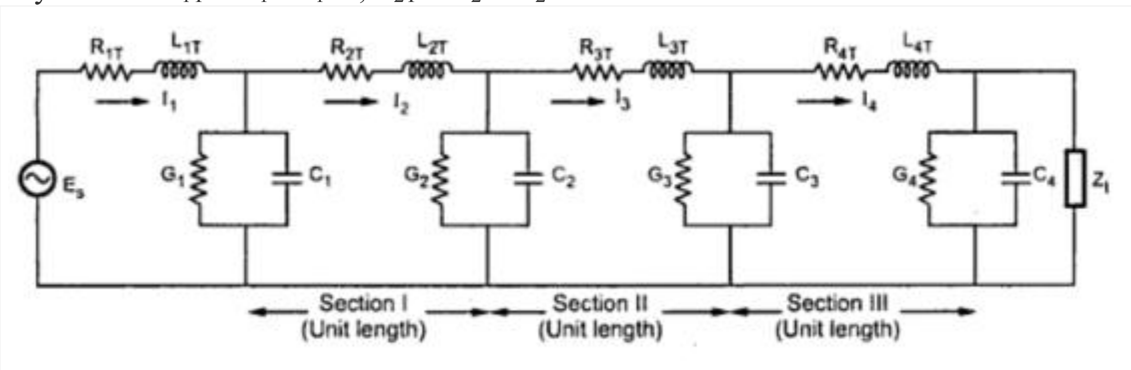
Consider the equivalent circuit of a 2-wire transmission line having resistance, inductance, capacitance and conductance associated with it as shown in the Fig. 4.



The line is divided into large number of small sections of unit length. The upper and lower part of each section carries same current. Hence we have the equivalent circuit as shown in the Fig. 5.

Here $R_{1T} = R_1 + R_1'$, $R_{2T} = R_2 + R_2'$

Similarly $L_{1T} = L_1 + L_1'$, $L_{2T} = L_2 + L_2'$ etc



The resistance per unit length of the line is say R where $R_{1T} = R_{2T} = R_{3T} = R_{4T} ... = R$, ohm/metre. We have $L_{1T} = L_{1T} = L_{1T} = ... = L$, henry/metre

$G_1 = G_2 = ... G$, mho/metre

$C_1 = C_2 = ... C$, farad/metre

In case of 2 wire transmission system the line parameters are expressed on loop basis while in case of 3 phase transmission system the parameters are represented on per conductor basis.

The physical length of the transmission line does not indicate that the line is short or long but it is based on comparison with the wavelength (λ) of the source.

Factor to be Considered while Deciding Transmission Line

Following are some of the important factors that need to be considered while deciding the transmission system.

1. Type and size of a conductor.
2. Voltage level.
3. Line regulation and control of voltage.
4. Efficiency of transmission.
5. Corona loss.
6. Power flow capability and stability
7. Requirement of compensation.
8. Levels of faults at various bus bars and requirement of new circuit breakers.
9. Grounding needs.
10. Protection schemes for new lines.
11. Co-ordination of insulation.
12. Mechanical design aspects which include stress and sag calculations, composition of conductor, spacing of conductor, and configurations for insulators.
13. Design of power system structure.
14. Economical aspects.

Resistance

The resistance of a transmission line is an important parameter as it is the main cause of power loss in a transmission line. It is defined as the opposition offered by the transmission line conductors to the flow of current. The resistance of the line is uniformly distributed along its whole length. But the performance of the line can be conveniently analaysed by considering the distributed resistance as lumped one as shown in the Fig. 1.

The effective resistance of a conductor is given by

$$R = \frac{\text{Power loss in conductor}}{I^2} \ (\Omega)$$

Where the power loss in conductor is in watts and current flowing through the conductor in amperes.

When the distribution of current in a conductor is uniform then the effective resistance is equal to d.c. resistance of the conductor. This direct resistance is given by,

$$R = \frac{\rho l}{a} \ (\Omega)$$

where ρ = resistivity of conductor (Ω-m)

l = length of conductor (m)

a = cross-sectional area of conductor (m^2)

The d.c. resistance computed from the above equation of the stranded conductors is greater as spiraling of the strands makes them longer than the conductor itself. This increase in resistance is taken as 1% for 3 strand conductors and 2% for concentrically stranded conductors.

Temperature is also one of the factor which will change the d.c. resistance of the metallic conductor. Its variation with resistance is as shown in the Fig. 1. It is seen to be linear over normal range of operation.

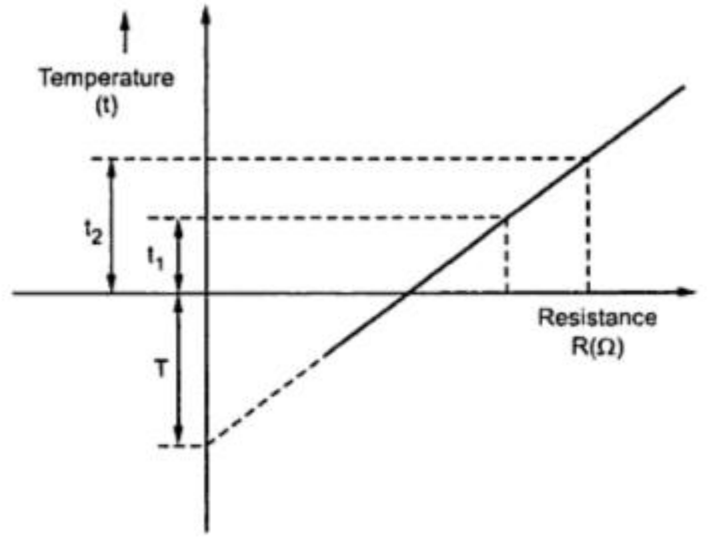


Fig. 1

From the above Fig. 1 it can be seen that

$$\frac{R_2}{R_1} = \frac{T+t_2}{T+t_1}$$

where R_1, R_2 are the resistances if the conductor at temperature t_1 and t_2 in ^oC and T is the constant obtained from the graph. The value of T is 234.5^{o}C for annealed copper 100% conductivity.

Inductance

An alternative flux is produced by the alternating current when flowing through a conductor. This flux links with the conductor. The conductor possesses inductance due to flux linkage. The flux linkages per ampere is called the inductance.

Thus inductance is given by

$$L = \psi/I$$

where ψ = Flux linkage in weber-turns

I = Current in amperes.

The inductance of a transmission line is also a distributed parameter over the length of line. For convenience in analysis it is taken as lumped as shown in the Fig. 1.

The fundamental equation used to define inductance is given by,

$$e = d\tau/dt \qquad \text{......................} \ (1)$$

where e is induced voltage in volts and is number of flux linkage of the circuit in weber-turns. The number of weber turns is the product of each weber of flux and the number of turns is the product of each weber of flux and the number of turns of the circuit linked. Each line of flux is multiplied by the number of turns it links and these products are added to determine total flux linkages.

If constant permeability is assumed for the medium in which the magnetic field is set up then we have

$N\tau \quad i$

$\therefore \quad N\tau = Li$

The constant of proportionality is called inductance

$\therefore \quad \tau = Li/N$

Substituting this value in the fundamental equation we have,

$$e = \frac{d}{dt}\left[\frac{Li}{N}\right] = \frac{L}{N}\frac{di}{dt}$$

For N number of turns,

$e = L\, di/dt$ .... (2)

If permeability is not constant then above equation may also be used but then the inductance is not constant.

Solving equations (1) and (2) we get,

$L = d\tau/dt$ H

With the flux linkage varying linearly with current then the magnetic circuit has constant permeability.

$L = \tau/i$ H

$\therefore \quad \tau = Li$ Wb-turns

In this equation, i is the instantaneous value of current. So represents instantaneous flux linkages. For Sinusoidal alternating current, flux linkages are also sinusoidal. Hence we have

$\psi = LI$

The voltage drop due to flux linkage is

$V = j\,\omega\, LI$ volts

$V = j\,\omega\,\psi$ volts

The mutual inductance between the two circuits is defined as the flux linkages of one circuit due to current in the second circuit per ampere of current in the second circuit per ampere of current in the second circuit.

If current produces ψ_{12} flux linkages with circuit 1, The mutual inductance is

$M_{12} = \psi_{12}/I_2$

The voltage drop in circuit 1 is given by

$V_2 = j\,\omega\, M_{12}\; I_2 = j\,\omega\,\psi_{12}$ volts

Mutual inductance is important in considering the influence of power lines on telephone lines and the coupling between parallel power lines.

Capacitance

When any two conductors are separated by an insulating material it will form a capacitor. In case of overhead transmission line air acts as an insulating medium between two conductors. So there will be capacitance between two conductors which is defined as charge per unit potential difference

Capacitance,

$C = q/v$ farad

Unlike resistance and inductance, the capacitance of transmission line is also uniformly distributed along the whole length of the line. It may be treated as a uniform series of capacitors connected between the conductors as shown in the Fig. 1 and Fig. 2.

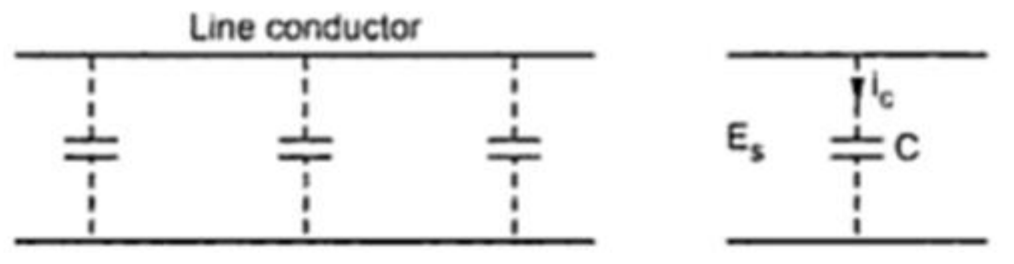


Fig. 1 Fig. 2

The charge on the conductors at any point increases with increase and decrease of the instantaneous value of the voltage between conductors at that point. Due to this a current known as charging current flows through the conductors even though the line is open circuited. The voltage drop, efficiency and power factor of line depends on the value of capacitance

Skin Effect

When a conductor carries a steady or d.c. current, this current is uniformly distributed over the whole cross-section of the conductor. However the current distribution is non-uniform if conductor carries alternating current. The current density is higher at the surface than at its centre. Thus the current is concentrated near the surface of the conductor as shown in the Fig. 1. This effect becomes predominent with increase in frequency. This behaviour of alternating current to concentrate near the surface of the conductor is known as skin effect.

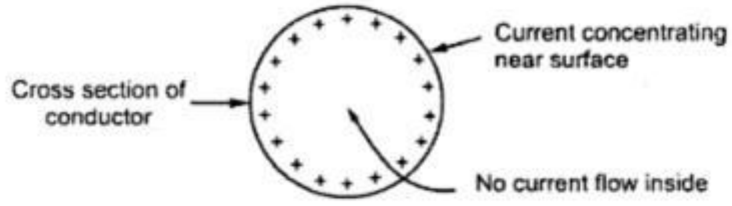


Fig. 1

Because of skin effect, larger power loss is caused for a given rms value of AC than the loss when the same value of DC is flowing through the conductor. Alternatively the effective resistance of conductor is more for AC than for DC.

Due to this skin effect, the effective cross sectional area offered to the flow of current decreases which increases resistance.

Consider a solid, round conductor consisting of large number of strands. Each strand is carrying a small part of current. The strands near the centre are surrounded by a greater magnetic flux and hence have large inductance than that near the surface. As we move towards the outer strands, the flux linking progressively reduces for the reason that the flux inside the strand does not link it. The reactance of inner strands is greater than outer strands which cause the alternating current to flow near the surface of the conductor. With increase in the frequency, The inductive reactance of the strands becomes and more non-uniform which leads to non-uniform current distribution.

The skin effect is quite significant for large, solid conductors even at a frequency of 50 Hz.

The skin effect depends on following factors

1. Nature of material
2. Diameter of wire
3. Frequency of supply
4. Shape of wire.

With increase in diameter of wire, the skin effect increases. Similarly as frequency increases, the skin effect increases. If we have stranded conductor rather than solid conductor then the skin effect is less. It can be seen that when supply frequency is less than 50 Hz and conductor diameter is less than 1 cm then skin effect is negligible. In large conductors at power frequencies the skin effect is a significant factor.

Proximity Effect

The current distribution may be non-uniform because of another effect known as proximity effect.

Consider a two wire line as shown in the Fig. 1.

Fig. 1

Let each of the line conductor is assumed to be divided into 3 sections having equal cross sectional area. Three parallel loops are formed by the pairs xx', yy' and zz'. The flux linking loop xx' is least and it increases for the remaining loops. Thus the inductance of inner loop is less. Thus the current density is highest at inner edges of the conductor. Due to this non-uniform distribution of current, the effective conductor resistance increases. As the distance between the conductors goes on reducing, this distribution of current becomes more and more non-uniform.

For normal spacing of overhead lines this proximity effect is negligible. For underground cables this effect is significant as the conductors are located close to each other.

The proximity effect also depends on the same factors as that of skin effect.

Concept of GMD and GMR

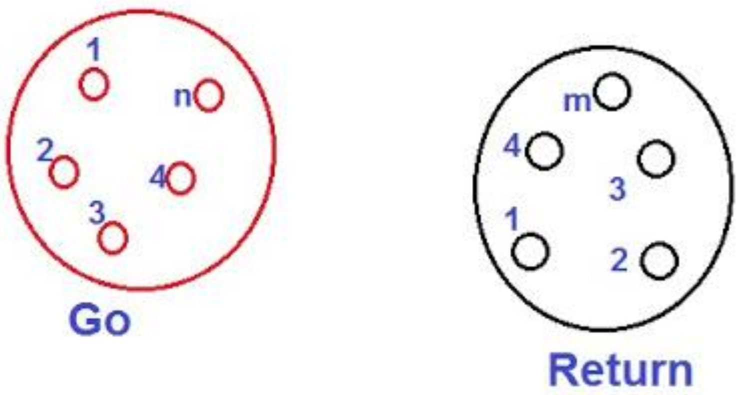
GMD & GMR stands for Geometrical Mean Distance and Geometrical Mean Radius. This concept is very useful in Power System for the calculation of Inductance and Capacitance of Transmission Line.

Basically, as we know that Geometrical Mean of n numbers $a_1, a_2, a_3, \ldots, a_n$ is

$(a_1 a_2 a_3 a_4 \ldots a_n)^{1/n}$

The same concept is also used for the calculation of GMD and GMR. In GMD we take the Geometrical Mean of distances between the strands of two Transmission Lines while in GMR, Geometrical Mean of distances between the stands of a single composite conductor are calculated.

Let us assume two composite conductors used in Transmission Line as shown in figure below.



As shown in the figure above, one conductor is Go and another is Return for current for single phase line. The current is assumed to be equally divided among all the strands of a conductor.

Therefore,

Current carried by each strand in Go conductor = I/n

Current carried by each strand in Return conductor = -I/m

Here I is the total current carried by each conductor.

Now, we will calculate the GMD and GMR for the configuration of the conductors shown in figure above.

For getting the GMD, first we need to calculate the distance between the strands of Go and Return conductors.

Let,

D11 = Distance between the 1st strand of Go and Return conductor

D12 = Distance between the 1st strand of Go and 2nd strand of Return conductor

D21 = Distance between the 2nd strand of Go and 1st strand of Return conductor

Dmn = Distance between the mth strand of Go and nth strand of Return conductor

Thus the Geometrical mean of above distances i.e. GMD,

$$= \sqrt[mn]{(D_{11}D_{12}\dots D_{1m})(D_{21}D_{22}\dots D_{2m})(D_{31}D_{32}\dots D_{3m})\dots\dots(D_{n1}D_{n2}\dots D_{nm})}$$

Geometrical Mean Radius of a solid conductor or a strand of radius R is defined as the factious radius R' having no internal flux linkage but having the same inductance as the original conductor of radius R.

R ' = 0.7788R

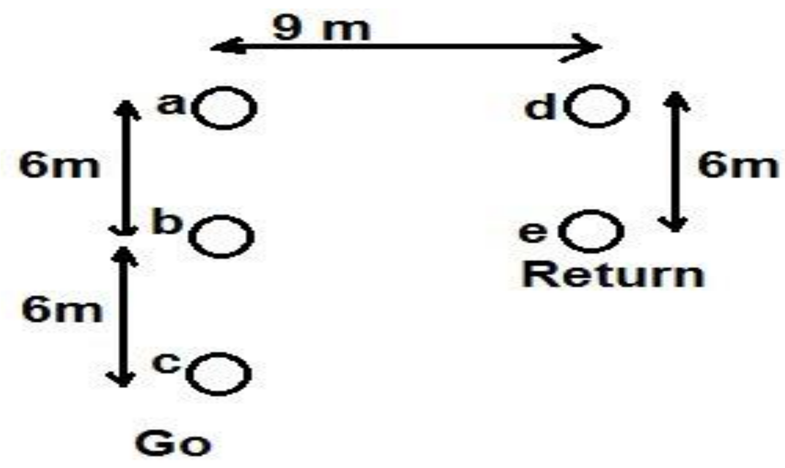
Method for Calculating GMR of a Composite Conductor:

For calculating GMR, first we find the distance between the individual strands. Thus if there are n strands in a composite conductor then obviously there will be n^2distances between the strands.

Thus GMR of Go conductor,

$$= \sqrt[n^2]{(R'D_{12}D_{13}\dots D_{1n})(R'D_{21}D_{23}\dots D_{2n})\dots\dots(R'D_{n1}D_{n2}\dots D_{(n-1)n})}$$

Let us now consider an example to make our concept clear. As shown in figure below, go conductor contain three strands of radius 2.5 mm while the return conductor contains two of radius 5mm.



GMR of individual strands in Go conductor R' = 0.7788xR

= 0.7788x2.5

= 1.947mm

GMR of Go Conductor

= $[(1.947x6x12)(1.947x6x6)(1.947x6x12)]^{1/9}$

= 0.4809m

Similarly,

GMR of individual strands in Return conductor R' = 0.7788xR

= 0.7788x5 mm

= 0.003894 m

Hence, GMR of Return Conductor

= $[(0.003894x6)(0.003894x6)]^{1/4}$

= 0.1528 m

Now, distance between strand a and e Dae = $[9^2+6^2]^{1/2}$

= 10.81 m

Distance between strand c and d, Dcd = $[9^2+12^2]^{1/2}$

= 15 m

Therefore,

GMD for the configuration,

= $[9x10.81x10.81x9x15x10.81]^{1/6}$ = 10.74 m

Inductance of a Conductor due to the Internal Flux

Consider a long, straight conductor with radius r meters and carrying a current I amperes as shown in the Fig.1(a).

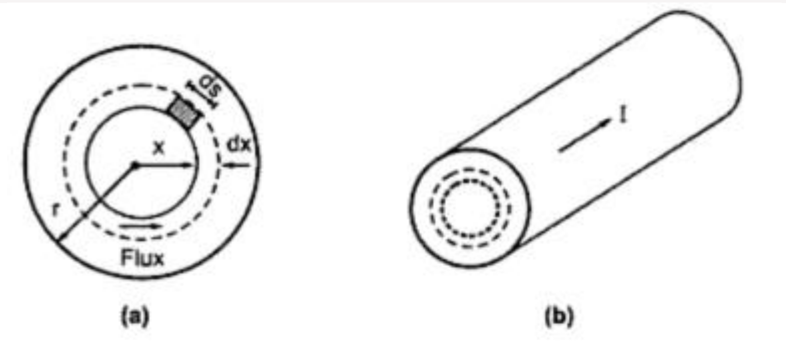


Fig. 1

The magnetic field will be established due to this current. The magnetic flux lines will change inside the conductors which will contribute to induced voltage and hence inductance. The magnetic flux lines exist outside the conductor also. We may assume that the return path for the current in this conductor is far away and the magnetic field of the conductor is not affected. The value of inductance due to internal flux is given by the ratio of flux linkages to current by taking into account the fact that each line of internal flux links only a fraction of total current. The exact value of inductance of transmission line is obtained by considering the flux inside each conductor as well as external flux. The lines of flux are concentric with the conductor.

The mmf in ampere turns around any closed path is equal to the current in amperes enclosed by the path. Thus we have

$$\text{mmf} = \oint H \cdot ds = I$$

where H is magnetic field intensity in At/m

Let the magnetic field intensity at a point x meters from the centre of the conductor be H_x. This is constant at all points as field is symmetrical. Thus integration of ds around the closed circular path is 2πx.

$$\oint ds = 2\pi x$$

As H_x is constant

$$H_x 2\pi x = I_x$$

$$\therefore \quad H_x = \frac{I_x}{2\pi x}$$

As the current density is uniform

$$\frac{I_x}{\pi x^2} = \frac{I_x}{\pi r^2}$$

$$\therefore \quad I_x = \frac{\pi x^2}{\pi r^2} I$$

$$\therefore \quad I_x = \frac{x^2}{r^2} I$$

Substituting this value of I_x in expression for H_x

$$H_x = \frac{x^2}{r^2} \cdot I \cdot \frac{1}{2\pi x} = \frac{x}{2\pi r^2} I \text{ AT/m}$$

If $\mu = \mu_o \mu_r$, then flux density at the given point is given by,

$$B_x = \mu H_x = \mu_0 \mu_r H_x$$
$$= \frac{\mu_0 \mu_r x}{2\pi r^2} I$$

$\mu_r = 1$ for the non-magnetic material

$$B_x = \frac{\mu_0 x}{2\pi r^2} \cdot I$$

For the element having thickness dx, the flux will be product of and the cross-sectional area of the element normal to the flux lines. This area is dx times axial length. If the axial length considered is 1m then the flux per meter of length is

$$d\phi = B_x \times 1 \times dx = \frac{\mu_0 x I}{2\pi r^2} dx$$

This flux links with current I_x. Hence flux linkage per meter length of conductor is given by,

$$d\psi = \frac{x^2}{r^2} d\phi = \frac{\mu_0 x^3 I}{2\pi r^4} dx$$

To find internal flux or the total flux linkage inside the conductor we have to carry the integration from the centre of conductor to its outside edge.

$$\psi_{int} = \int_0^r d\psi = \int_0^r \frac{\mu_0 x^3 I}{2\pi r^4} \cdot dx$$
$$\psi_{int} = \frac{\mu_0 I}{8\pi} \quad \text{Wb-turn/m}$$

But μ_o = permeability of free space = $4\pi \times 10^{-7}$ H/m

$$\psi_{int} = \frac{I}{2} \times 10^{-7} \text{Wb-t/m}$$
$$\therefore \quad L_{int} = \frac{\psi_{int}}{I} = \frac{10^{-7}}{2} \text{ H/m}$$

Thus we have obtained inductance per unit length of a round conductor due to flux inside the conductor. Inductance per unit length is referred as simply inductance for convenience and simplicity.

Inductance of a Conductor due to External Flux

Now we shall estimate the flux linkages of the conductor due to the external flux. For this we will consider the flux linkages of an isolated conductor due to that portion of the external flux which lies between two points distant and meters from centre of conductor P_1 and P_2 are two such points as shown in the Fig. 1.

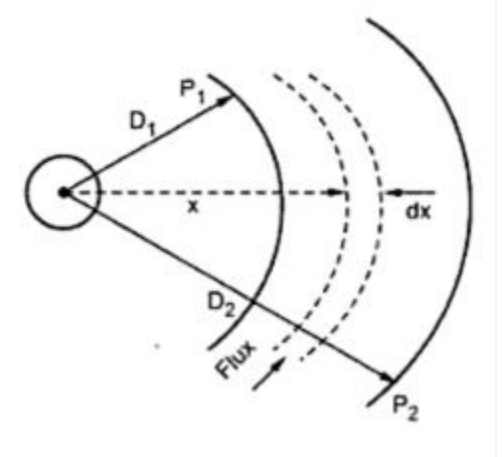


Fig. 1

The conductor shown in the Fig. 1 carries current I. The flux paths are concentric circles around the conductor between P_1 and P_2.

Consider a tubular element which is x meters from centre of conductor. The field intensity at this point is H_x. The mmf around the element is

$$2\pi x\ H_x = I$$

The flux density B_x at this point is given by

$$B_x = \mu H_x = \frac{\mu I}{2\pi x}\ Wb/m^2$$

The flux dΦ in the tubular element is given by,

$d\Phi = B_x$ x dx x 1m (Axial length is considered as 1m)

$$\therefore \qquad d\phi = \frac{\mu I}{2\pi x}\cdot dx$$

The flux linkages dψ per meter equal to dΦ since flux external to the conductor links all the current in the conductor. The total flux linkage between P_1 and P_2 are obtained by integrating dψ from D_1 to D_2

$$\psi_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x}\, dx$$

$$= \frac{\mu I}{2\pi}\int_{D_1}^{D_2} \frac{dx}{x}$$

$$= \frac{\mu I}{2\pi}\, ln\left(\frac{D_2}{D_1}\right)$$

Since the $\mu = \mu_o\ \mu_r$

For Relatively permeability, $\mu_r = 1$

$$\psi_{12} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)$$

$$= \frac{4\pi \times 10^{-7} I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)$$

$$\psi_{12} = 2 \times 10^{-7}\, I \ln\left(\frac{D_2}{D_1}\right)$$

The inductance due to flux included between P_1 and P_2 only is,

$$L_{12} = \frac{\psi_{12}}{I} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right)$$

In the external flux is considered to be extended from the surface of conductor to infinity then total flux linkages is given by,

$$\psi_{12} = \int_r^{\infty} \frac{\mu_0 I}{2\pi x}\, dx \quad \text{weber-turns}$$

Overall flux linkages is given by

$$\text{Total flux linkage} = \frac{\mu_0 I}{8\pi} + \int_r^{\infty} \frac{\mu_0 I}{2\pi x}\, dx$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right]$$

Inductance of a Single Phase Two Wire Line

Consider a single phase line consisting of two parallel conductors. These conductors are forming a rectangular loop of one turn. These conductors are solid conductors of radius r_1 and r_2 respectively. One conductor is forming a return circuit for the other. The two conductors are carrying currents I_1 and I_2 respectively.

In a single phase circuit we have

$$I_1 + I_2 = 0$$

$$\therefore \quad I_2 = -I_1$$

Here we are neglecting the effect of earth's presence of magnetic field geometry as earth's relative permeability is same as that of air and its conductivity is relatively small.

The arrangement of the conductors and variation of flux density due to each conductor is shown in the Fig. 1 and Fig. 2 respcetively.

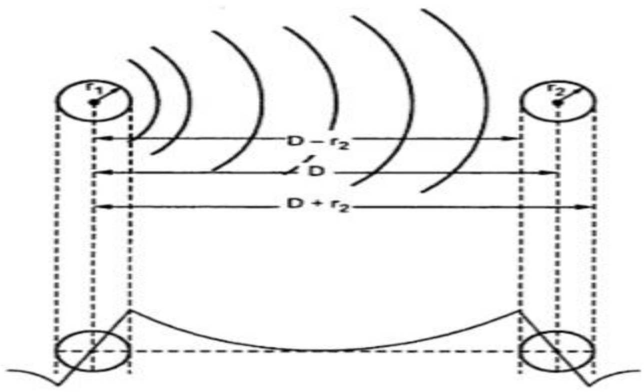


Fig. 1 and Fig. 2

In the beginning, let us consider only the flux linkages of the circuit caused by the current in conductor 1. The flux line set up by the current flowing in conductor 1 at a distance equal to or greater than D+ r_2 from the centre of conductor 1 does not link the circuit and hence is not responsible for inducing and voltage in the circuit. This is because conductor 2 carries current which is equal and opposite to that in conductor 1.

The external flux from r_1 to D - r_2 links all the current I_1 in conductor 1. Over the surface of conductor 2 i.e. between (D - r_2) and (D + r_2), the external flux links a current whose magnitude is progressively reduces from I_1 to zero because of negative current in conductor 2.

The total inductance of the current carrying in conductor 1 can be calculated by assuming that D is much greater than r_1 and r_2. Under this condition it can be assumed that flux from (D - r_2) to the centre of conductor 2 links I_1 current and flux from the centre of conductor 2 to (D + r_2) links zero current.

The inductance due to current in conductor 1 can be calculated by using the relation,

$$L_{12} = 2 \times 10^{-7}\, ln\left(\frac{D_2}{D_1}\right) H/m$$

$$\therefore \quad L_{12} = 2 \times 10^{-7}\, ln\left(\frac{D}{r_1}\right) H/m$$

This expression is valid for external flux only. For internal flux we have

$$L_{1\,int} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

The total inductance of the circuit due to current in conductor 1 only is,

$$L_1 = L_{1\,int} + L_{12} = \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7}\, ln\left(\frac{D_2}{r_1}\right)$$

$$= \left[\frac{1}{2} + 2\,ln\left(\frac{D}{r_1}\right)\right] 10^{-7} \text{ H/m} \qquad \ldots (1)$$

The above equation can conveniently be written as,

$$L_1 = 2 \times 10^{-7}\left[ln\varepsilon^{1/4} ln\left(\frac{D}{r_1}\right)\right]$$

where $\frac{1}{4} = ln\,\varepsilon^{1/4}$

$$L_1 = 2 \times 10^{-7}\, ln\left[\frac{\varepsilon^{1/4} D}{r_1}\right] = 2 \times 10^{-7}\, ln\left[\frac{D}{\varepsilon^{-1/4} r_1}\right]$$

Let us consider $r_1' = r_1\, \varepsilon^{-1/4}$ then we have

$$L_1 = 2 \times 10^{-7}\, ln\left[\frac{D}{r_1'}\right] \qquad \text{... (2)}$$

The radius r_1' is that of an imaginary or fictitious conductor assumed to have no internal flux. The quantity $\varepsilon^{-1/4}$ equals to 0.7778.

The value of inductance given by equation (2) is same as that given by equation (1). The difference is that equation (2) omits the term on account of internal flux. But it is compensated by adjusted value of radius of conductor.

The above equation is derived by considering solid round conductors. Equation II is algebraic manipulation of equation (1). Hence the multiplying factor of 0.7778 is applicable only to solid round conductors in order to account for internal flux.

The conductor 2 carries current in opposite direction to that in conductor 1. The flux linkages produced by current in conductor 2 considered alone are in the same direction as those produced by current in conductor 1.

The resultant flux for the two conductors is determined by sum of mmfs of the two conductors. If permeability is assumed to be constant then the flux linkages and inductances of the two conductors calculated separately may be added.

The inductance of conductor 2 in comparison with equation (2) can be written as,

$$L_2 = 2 \times 10^{-7}\, ln\left[\frac{D}{r_2'}\right] \text{ H/m}$$

For the total circuit

$$L = L_1 + L_2 = 4 \times 10^{-7}\, ln\left[\frac{D}{\sqrt{r_1' r_2'}}\right] \text{ H/m}$$

If we have $r_1' = r_2' = r'$

Total inductance, $L = 4 \times 10^{-7}\, ln\left[\frac{D}{r'}\right]$ H/m

The above equation gives the inductance of two wire single phase line taking into consideration the flux linkages caused by current in both the conductors.

The value of inductance obtained is the inductance per loop meter or per loop mile. The inductance given by equation (2) is one half of the total inductance of single phase line and is called inductance per conductor.

Inductance of Composite Conductor Lines

We will consider a single phase 2 wire system. It consists of two conductors say P and Q which are composite conductors. The arrangement of conductors is shown in the Fig. 1.

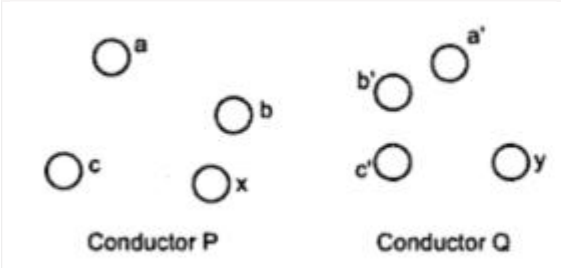


Fig. 1

Conductor P is consisting of x identical, parallel filaments. Each of the filament carries a current of I/x. Conductor Q consists of y filament with each filament carrying a current of - I/y. The conductor Y carries a current of I amps in opposite direction to the current in conductor X as it is forming return path.

The flux linkages of filament say a due to all currents in all the filaments is given by

$$\psi_a = 2\times10^{-7}\left[\frac{I}{x}ln\frac{1}{D_{aa}}+\frac{I}{x}ln\frac{1}{D_{ab}}+\frac{I}{x}ln\frac{1}{D_{ac}}+....+\frac{I}{x}ln\frac{1}{D_{ax}}\right]$$

$$+2\times10^{-7}\left[\frac{-I}{y}ln\frac{1}{D_{aa'}}+\frac{-I}{y}ln\frac{1}{D_{ab'}}+\frac{-I}{y}ln\frac{1}{D_{ac'}}+....+\frac{-I}{y}ln\frac{1}{D_{ay}}\right]$$

$$= 2\times10^{-7}\frac{I}{x}\left[ln\frac{1}{D_{aa}}+ln\frac{1}{D_{ab}}+....+ln\frac{1}{D_{ax}}\right]$$

$$-2\times10^{-7}\frac{I}{y}\left[ln\frac{1}{D_{aa'}}+ln\frac{1}{D_{ab'}}+....+ln\frac{1}{D_{ay}}\right]$$

$$= 2\times10^{-7}I\left[ln\left(\frac{1}{D_{aa}}\right)^{1/x}+ln\left(\frac{1}{D_{ab}}\right)^{1/x}+....+ln\left(\frac{1}{D_{ax}}\right)^{1/x}\right]$$

$$-2\times10^{-7}I\left[ln\left(\frac{1}{D_{aa'}}\right)^{1/y}+ln\left(\frac{1}{D_{ab'}}\right)^{1/y}+....+ln\left(\frac{1}{D_{ay}}\right)^{1/y}\right]$$

$$= 2\times10^{-7}I\left[ln\frac{\sqrt[y]{D_{aa'}\cdot D_{ab'}....D_{ay}}}{\sqrt[x]{D_{aa}D_{ab}....D_{ax}}}\right]$$

The inductance of filament a is given by,

$$L_a = \frac{\psi_a}{I/x} = 2x\times10^{-7}\,ln\left[\frac{\sqrt[y]{D_{aa'}\cdot D_{ab'}....D_{ay}}}{\sqrt[x]{D_{aa}D_{ab}....D_{ax}}}\right]$$

The inductance of filament b is given by,

$$L_b = \frac{\psi_b}{I/x} = 2x\times10^{-7}\,ln\left[\frac{\sqrt[y]{D_{ba'}\cdot D_{bb'}....D_{by}}}{\sqrt[x]{D_{ba}D_{bb}....D_{bx}}}\right]$$

The average inductance of the filaments of conductor P is

$$L_{av} = \frac{L_a + L_b + L_c + \ldots + L_x}{x}$$

The conductor P consists of x number of parallel filaments. If all the filaments are equal inductances then inductance of the conductor would be 1/x times inductance of one filament. All the filaments have different inductances but the inductance of all of them in parallel is 1/x times the average inductance.

Inductance of conductor P is given by,

$$L_p = \frac{L_{av}}{x} = \frac{L_a + L_b + L_c \ldots + L_x}{x^2}$$

Substituting the values of L_a, L_b ..... L_x in the equation and simplifying the expression we have,

$$L_p = 2 \times 10^{-7}\, ln\left[\frac{\sqrt[xy]{(D_{aa'}D_{ab'} \ldots D_{ay})(D_{ba'}D_{bb'} \ldots D_{by})\ldots(D_{xa'} \ldots D_{xy})}}{\sqrt[x^2]{(D_{aa}D_{ab} \ldots D_{ax})(D_{ba}D_{bb} \ldots D_{bx})\ldots(D_{xa}D_{xb} \ldots D_{xx})}}\right]$$

In the above expression the numerator of argument of logarithm is the xy, the root of xy terms. These terms are nothing but products of distance from all the x filaments of conductor P to all the y filaments of the conductor Q.

For each filament in conductor P there are y distances to filaments in conductor Q and there are x filaments in conductor P. The xy terms are formed as a result of product of y distances for each of x filaments. The xyth root of the product of the xy distances is called the geometric mean distance between conductor P and Q. It is termed as D_m or GMD and is called mutual GMD between the conductors.

The denominator of the above expression is the x^2 root of x^2 terms. There are x filaments and for each filament there are x terms consisting of r' (denoted by D_{aa}, D_{bb} etc) for that filament times the distance from that filament to every other filament in conductor P.

If we consider the distance D_{aa} then it is the distance of the filament from itself which is also denoted as r_a'. This r' of a separate filament is called the self GMD of the filament. It is also called geometric mean radius GMR and identified as D_s.

Thus the above expression now becomes

$$L_p = 2 \times 10^{-7}\, ln\left[\frac{D_m}{D_s}\right] H/m$$

Comparing this equation with the expression obtained for inductance of a single phase two wire line. The distance between solid conductors of single conductors line is substituted by the GMD between conductors of the composite conductor line. Similarly the GMR (r') of the single conductor is replaced by GMR of composite conductor.

The composite conductors are made up of number of strands which are in parallel. The inductance of composite conductor Q is obtained in a similar manner. Thus the inductance of the line is,

$$L = L_p + L_Q$$

An Alternative Approach for Finding the Expression for Inductive Reactance

For standard conductors generally the values for GMR are available. These tables are also useful for calculating inductive reactance as well as shunt capacitive reactance and resistance. The units used in these tables are inches, feet and miles as used by the industries in the united states.

Normally value of inductive reactance is required rather than the inductance. The inductive reactance of one conductor of a single phase two conductor is given by,

$$X_L = 2\pi f L = 2 \times 10^{-7} \ln\left(\frac{D_m}{D_s}\right) \times 2\pi f$$

$$= 4\pi f \times 10^{-7} \ln\left(\frac{D_m}{D_s}\right)$$

$$\therefore \quad X_L = 2.022 \times 10^{-7} f \ln\left(\frac{D_m}{D_s}\right)$$

The GMR found from the tables is an equivalent D which takes into account skin effect which affects the value of inductance appreciably.

Expanding the above equation we have,

$$X_L = 2.022 \times 10^{-3} f \ln\left(\frac{1}{D_s}\right) + 2.022 \times 10^{-3} f \ln D_m$$

Considering both D_m and D_s to be expressed in feets, the first term in the equation is the inductive reactance of one conductor of a two conductor line having a distance of 1ft between them. Hence the first term is called inductive reactance at 1ft spacing X_a. It depends upon the GMR of the conductor and frequency. The second term is called inductive reactance spacing factor X_d. This term is independent of type of conductor and depends only on frequency and spacing.

When $D_m = 1$, the spacing factor is equal to zero. When it is less than 1, the spacing factor value is negative. The inductive reactance at 1ft spacing value is available from the table. To that value, the value of inductive reactance spacing factor is added which is also available at desired line frequency.

Inductance of Three Phase Lines With Equilateral and Symmetrical Spacing

Consider a three phase line consisting of three phase conductors a, b and c as shown in the Fig. 1. These three conductors are equally spaced at the corners of an equilateral triangle having radius r.

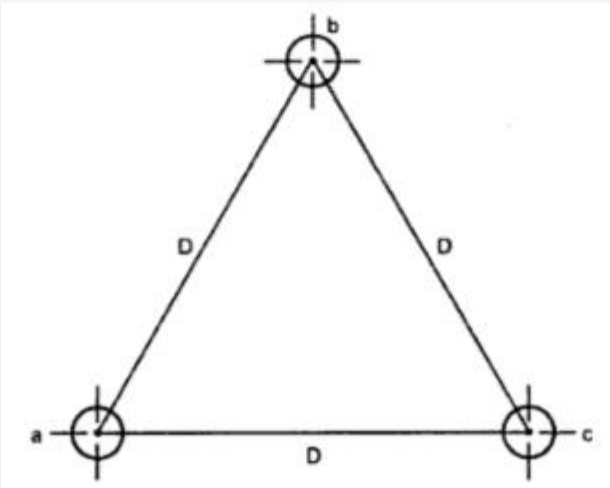


Fig. 1

The flux linkages of conductor a are given by,

$$\psi_a = 2 \times 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right)$$

If the currents are assumed to be balanced than

$$I_a + I_b + I_c = 0$$

$\therefore \quad I_a = -(I_b + I_c)$

or $\quad (I_b + I_c) = -I_a$

The above equation becomes,

$$\psi_a = 2 \times 10^{-7}\left(I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D}\right)$$

$$= 2 \times 10^{-7} I_a\left(\ln\frac{1}{r'} - \ln\frac{1}{D}\right) = 2 \times 10^{-7} I_a \ln\left(\frac{D}{r'}\right)$$

The inductance of conductor a is given by

$$L_a = \frac{\psi_a}{I_a} = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right)$$

Hence $\quad L_a = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right)$

because of symmetry, conductors b and c will have same inductance as that of conductor a. Each phase consists of only one conductor. So the above equation gives inductance per phase of the three phase lines. For stranded conductor we write D_s whereas for single conductor it is replaced by r'.

Inductance of Transmission Line With Unsymmetrical Spacing

Consider the same three conductors a, b, c having radius r but Unsymmetrically speed as shown in the Fig. 1.

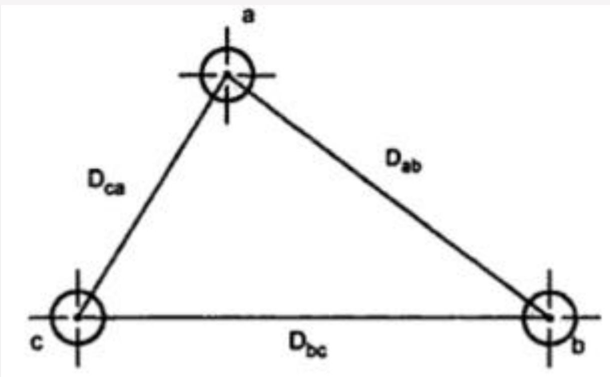


Fig. 1

Flux linkage of conductor a is given by,

$$\psi_a = 2\times 10^{-7}\left[I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ca}}\right]$$

$$\text{Inductance of conductor a, } L_a = \frac{\psi_a}{I_a} = 2\times 10^{-7}\left[\ln\frac{1}{r'} + \frac{I_b}{I_a}\ln\frac{1}{D_{ab}} + \frac{I_c}{I_a}\ln\frac{1}{D_{ca}}\right]$$

If all the three currents are assumed to be balanced with I_a as a reference phasor.

$$\bar{I}_a = I_a\angle 0^\circ;\ \bar{I}_b = I_b\angle 240^\circ = I_a\angle 240^\circ;\ \bar{I}_c = I_c\angle 120^\circ = I_a\angle 120^\circ$$

$$\frac{I_b}{I_a} = 1\angle 240^\circ = \cos 240^\circ + j\sin 240^\circ = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\frac{I_c}{I_a} = 1\angle 120^\circ = \cos 120^\circ + j\sin 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Substituting this in equation for L_a,

$$L_a = 2\times 10^{-7}\left[\ln\frac{1}{r'} + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\ln\frac{1}{D_{ab}} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\ln\frac{1}{D_{ca}}\right]$$

$$= 2\times 10^{-7}\left[\ln\frac{1}{r'} + \ln\left(\sqrt{D_{ab}D_{ca}}\right) + j\sqrt{3}\ln\sqrt{\frac{D_{ab}}{D_{ca}}}\right]$$

Similarly we have,

$$\psi_b = 2\times 10^{-7}\left[I_a\, ln\frac{1}{D_{ab}} + I_b\, ln\frac{1}{r'} + I_c\, ln\frac{1}{D_{bc}}\right]$$

$$L_b = 2\times 10^{-7}\left[ln\frac{1}{r'} + \frac{I_a}{I_b} ln\frac{1}{D_{ab}} + \frac{I_c}{I_b} ln\frac{1}{D_{bc}}\right]$$

$$L_b = 2\times 10^{-7}\left[ln\frac{1}{r'} + (1\angle -240^\circ)\, ln\frac{1}{D_{ab}} + (1\angle -120^\circ)\, ln\frac{1}{D_{bc}}\right]$$

$$= 2\times 10^{-7}\left\{ln\frac{1}{r'}\left[-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right] ln\frac{1}{D_{ab}} + \left[-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right] ln\frac{1}{D_{bc}}\right\}$$

$$= 2\times 10^{-7}\left\{ln\frac{1}{r'} + ln\sqrt{D_{ab}D_{bc}} + j\sqrt{3}\, ln\sqrt{\frac{D_{bc}}{D_{ab}}}\right\}$$

Also we have,

$$\psi_c = 2\times 10^{-7}\left[I_a\, ln\frac{1}{D_{ca}} + I_b\, ln\frac{1}{D_{bc}} + I_c\, ln\frac{1}{r'}\right]$$

Which on simplifying gives,

$$\psi_c = 2\times 10^{-7}\left[ln\frac{1}{r'} + ln\sqrt{D_{ca}D_{bc}} + j\sqrt{3}\, ln\sqrt{\frac{D_{ca}}{D_{bc}}}\right]$$

The inductances of the three phases are respectively as follows,

$$L_a = 2\times 10^{-7}\left[ln\frac{1}{r'} + ln\sqrt{D_{ab}D_{ca}} + j\sqrt{3}\, ln\sqrt{\frac{D_{ab}}{D_{ca}}}\right]$$

$$L_b = 2\times 10^{-7}\left[ln\frac{1}{r'} + ln\sqrt{D_{ab}D_{bc}} + j\sqrt{3}\, ln\sqrt{\frac{D_{bc}}{D_{ab}}}\right]$$

$$L_c = 2\times 10^{-7}\left[ln\frac{1}{r'} + ln\sqrt{D_{ca}D_{bc}} + j\sqrt{3}\, ln\sqrt{\frac{D_{ca}}{D_{bc}}}\right]$$

From the above expressions it can be seen that the individual phase inductance of a line which is Unsymmetrically spaced is a complex number. The imaginary part in the expression for inductance represents exchange of energy between phases.

Inductance of Three Phase Double Circuit with Symmetrical Spacing

Consider the three phase double circuit with the conductor placed at the vertices of a regular hexagon which is shown in the Fig. 1.

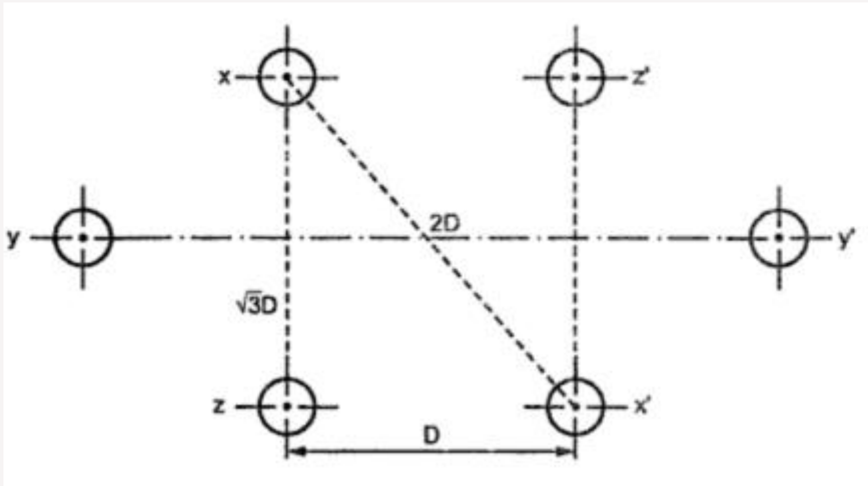


Fig. 1

Conductor x, y, z are forming one circuit whereas conductors x' , y' and z' are forming another circuit. Let us find out flux linkage of conductor x due to currents in other phases.

$$\psi_x = 2\times10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{2D}\right)+I_y\left(ln\frac{1}{D}+ln\frac{1}{\sqrt{3}D}\right)+I_z\left(ln\frac{1}{\sqrt{3}D}+ln\frac{1}{D}\right)\right\}$$

$$= 2\times10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{2D}\right)+(I_y+I_z)\left(ln\frac{1}{D}+ln\frac{1}{\sqrt{3}D}\right)\right\}$$

since the $I_x + I_y + I_z = 0$

$\therefore$ $I_y + I_z = - I_x$

$$= 2\times10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{2D}\right)-I_x\left(ln\frac{1}{D}+ln\frac{1}{\sqrt{3}D}\right)\right\}$$

$$= 2\times10^{-7}\left\{I_x\left[ln\left(\frac{1}{2Dr'}\right)\right]-I_x\left[ln\left(\frac{1}{\sqrt{3}D^2}\right)\right]\right\}$$

$$= 2\times10^{-7}\, I_x\, ln\left(\frac{\sqrt{3}D^2}{2Dr'}\right)$$

$$\psi_x = 2\times10^{-7}\, I_x\, ln\left(\frac{\sqrt{3}D}{2r'}\right)$$

$$L_x = \frac{\psi_x}{I_x} = 2\times10^{-7}\; ln\left(\frac{\sqrt{3}D}{2r'}\right)\text{H/m}$$

As the conductor are in parallel,

Inductance of each conductor = $2L_x$

$$= 4\times10^{-7}\, ln\left(\frac{\sqrt{3}D}{2r'}\right)\text{H/m}$$

Inductance of Three Phase Lines with Unsymmetrical Spacing but Transposed

Consider a three phase line having three conductors but not spaced equilaterally. The problem of finding the inductance in this case is difficult. The flux linkage and the corresponding inductance will not be same in each phase. Due to this different inductance per phase there is unbalance in the circuit through the currents in each phases are balanced.

The drops in the three phases due to these inductance are observed to be different. Thus at the receiving end we will not get the same voltage.

In order to achieve balance under this case, transposition of transmission line is preferred after a certain fixed distance. This is shown in the Fig. 1.

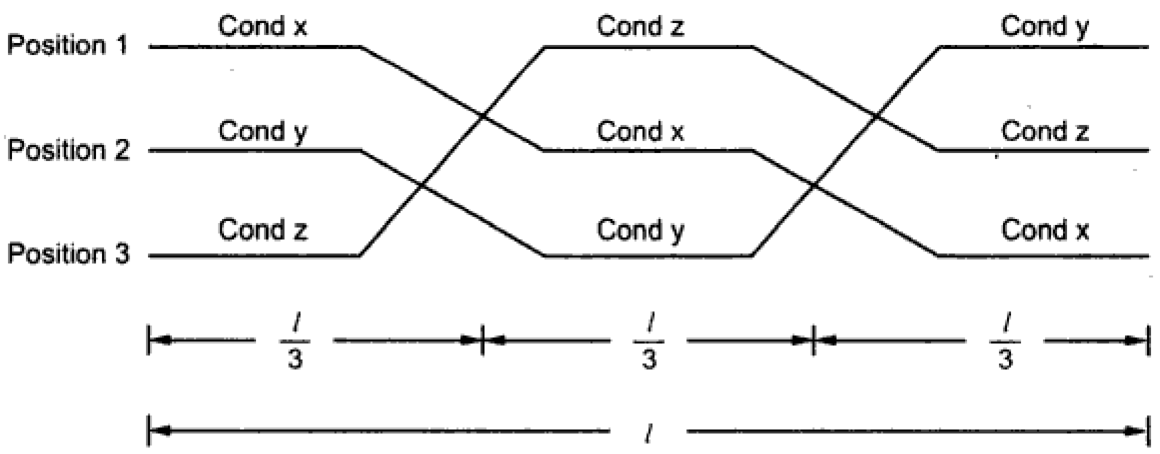


Fig. 1

The positions of the conductors are exchanged at regular interval along the line so that each conductor occupies the original position of every other conductor over an equal distance. This exchange of conductor positions is called transposition. Thus balance in the three phase is restored.

The Fig. 2 shows complete transposition cycle. The conductors in the individual phases are denoted by x, y and z where the positions are given by 1, 2, and 3. The same average inductance over the complete cycle is obtained due to the transposition.

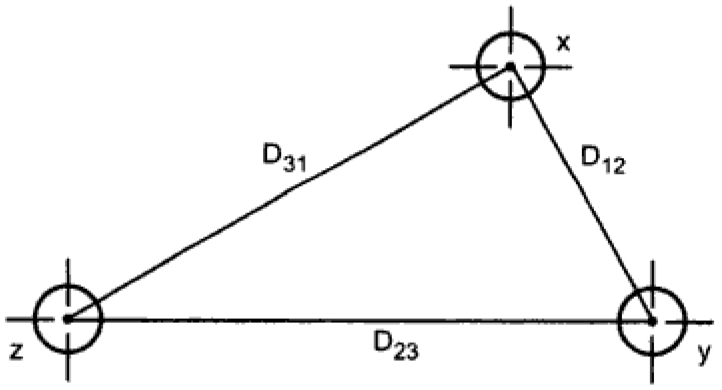


Fig. 2

The average inductance of one conductor is obtained by finding the flux linkages of a conductor for each position that is occupied during a complete cycle of transposition. Then the average flux linkages are obtained.

Now let us find the flux linkages of conductor x which is in position 1 whereas conductor y and z are in positions 2 and 3 respectively.

$$\psi_{x1} = 2\times 10^{-7}\left(I_x \ln\frac{1}{r'} + I_y \ln\frac{1}{D_{12}} + I_z \ln\frac{1}{D_{31}}\right)$$

Conductor x is in position 2 whereas conductors y and z are in positions 3 and 1 respectively.

$$\psi_{x2} = 2\times 10^{-7}\left(I_x \ln\frac{1}{r'} + I_y \ln\frac{1}{D_{23}} + I_z \ln\frac{1}{D_{12}}\right)$$

Conductor x is in position 3 whereas conductor y and z are in position 1 and 2 respectively.

$$\psi_{x3} = 2\times 10^{-7}\left(I_x \ln\frac{1}{r'} + I_y \ln\frac{1}{D_{31}} + I_z \ln\frac{1}{D_{23}}\right)$$

The average value of flux linkage of conductor x is

$$\psi_x = \frac{\psi_{x1}+\psi_{x2}+\psi_{x3}}{3}$$

$$\psi_x = \frac{2\times10^{-7}}{3}\left(3 I_x \ln\frac{1}{r'} + I_y \ln\frac{1}{D_{12}D_{23}D_{31}} + I_z \ln\frac{1}{D_{12}D_{23}D_{31}}\right)$$

$$\psi_x = \frac{2\times10^{-7}}{3}\left[3 I_x \ln\frac{1}{r'} + (I_y + I_z)\ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

But we have $I_x + I_y + I_z = 0$

$\therefore$ $I_y + I_z = -I_x$

$$\therefore \quad \psi_x = \frac{2\times10^{-7}}{3}\left[3 I_x \ln\frac{1}{r'} - I_x \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$\therefore \quad \psi_x = 2\times 10^{-7} I_x \ln\left[\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right]$$

Thus the average inductance per phase is

$$L_x = 2\times 10^{-7} \ln\left[\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right]$$

$$L_x = 2\times 10^{-7} \ln\left[\frac{D_{eq}}{r'}\right]$$

where $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

In modern power lines, transposition of lines is not done at regular intervals even though an exchange in conductor positions can be made at switching stations to balance the inductance per phase. The inequality in the phases of an untransposed line is small and neglected in many cases.

If the dissymmetry is neglected, the inductance of the untransposed line is the average value of the inductive reactance of one phase of the same line correctly transposed.

Inductance of Three Phase Double Circuit with Unsymmetrical Spacing but Transposed

Consider three phase double circuit having conductors which are Unsymmetrically spaced. The line is transposed so that each conductor occupies the position of other conductor after certain interval length or distance of transmission line.

The transposition cycle is as shown in the Fig.1.

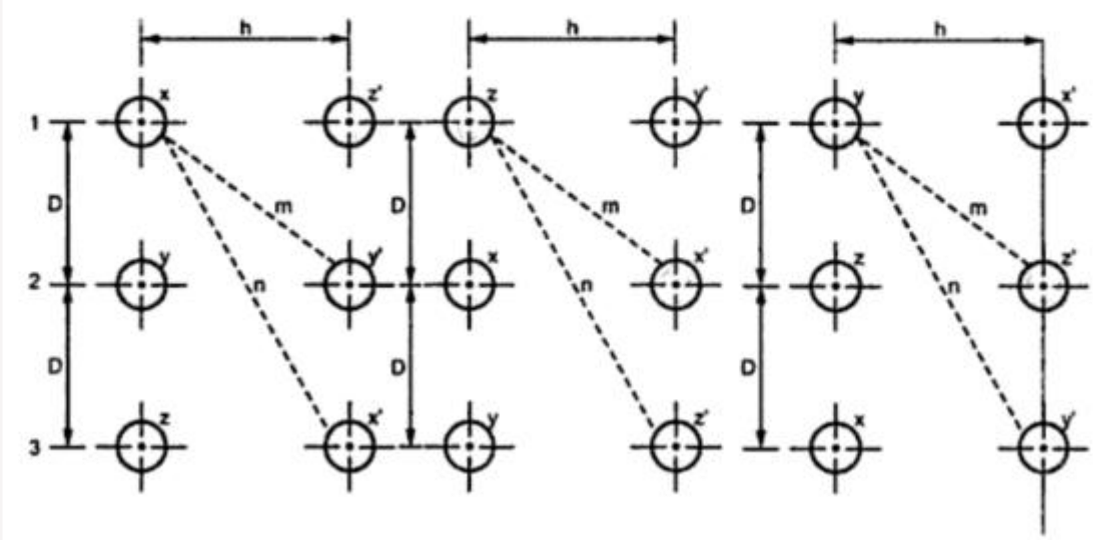


Fig. 1

Flux linkage of conductor x in position 1

$$\psi_{x1} = 2\times 10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{n}\right)+I_y\left(ln\frac{1}{D}+ln\frac{1}{m}\right)+I_z\left(ln\frac{1}{2D}+ln\frac{1}{h}\right)\right\}$$

Similarly,

$$\psi_{x2} = 2\times 10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{h}\right)+I_y\left(ln\frac{1}{D}+ln\frac{1}{m}\right)+I_z\left(ln\frac{1}{D}+ln\frac{1}{m}\right)\right\}$$

$$\psi_{x3} = 2\times 10^{-7}\left\{I_x\left(ln\frac{1}{r'}+ln\frac{1}{n}\right)+I_y\left(ln\frac{1}{2D}+ln\frac{1}{h}\right)+I_z\left(ln\frac{1}{D}+ln\frac{1}{m}\right)\right\}$$

We have, $\psi_x = \frac{1}{3}(\psi_{x1}+\psi_{x2}+\psi_{x3})$

$$\psi_x = \frac{2\times10^{-7}}{3}\left\{\begin{matrix}3I_x\,ln\frac{1}{r'}+I_x\,ln\left(\frac{1}{n}\cdot\frac{1}{h}\cdot\frac{1}{n}\right)+I_y\,ln\left(\frac{1}{2D^3}\right)\\ +I_y\,ln\left(\frac{1}{m}\cdot\frac{1}{m}\cdot\frac{1}{h}\right)+I_z\,ln\left(\frac{1}{2D^3}\right)+I_z\,ln\left(\frac{1}{h}\cdot\frac{1}{m}\cdot\frac{1}{m}\right)\end{matrix}\right\}$$

$$= \frac{2\times10^{-7}}{3}\left\{3I_x\,ln\frac{1}{r'}+I_x ln\left(\frac{1}{n^2h}\right)+(I_y+I_z)\,ln\left(\frac{1}{2D^3}\right)+(I_y+I_z)\,ln\left(\frac{1}{m^2h}\right)\right\}$$

As $I_x+I_y+I_z=0$ $\therefore$ $I_x = -(I_y+Iz)$

$$= \frac{2\times10^{-7}}{3}\left\{3\,I_x\,ln\frac{1}{r'}+I_x\,ln\left(\frac{1}{n^2h}\right)-I_x\,ln\left(\frac{1}{2D^3}\right)-I_x\,ln\left(\frac{1}{m^2h}\right)\right\}$$

$$= \frac{2\times10^{-7}}{3}\,I_x\left\{ln\left(\frac{2D^3m^2h}{3n^2h}\right)\right\}$$

$$= 2\times10^{-7}\,I_x\left\{ln\frac{2^{1/3}Dm^{2/3}h^{1/3}}{r'n^{2/3}h^{1/3}}\right\}$$

$$L_x = \frac{\psi_x}{I_x} = 2\times10^{-7}\,ln\left[2^{1/3}\left(\frac{D}{r'}\right)\left(\frac{m}{n}\right)^{2/3}\right]\ \text{H/m}$$

Inductance of each phase = 0.5 Inductance per conductor

$$= \frac{1}{2}L_x$$

$$= 2\times10^{-7}\,ln\left[2^{1/6}\left(\frac{D}{r'}\right)^{1/2}\left(\frac{m}{n}\right)^{1/3}\right]\ \text{H/m}$$

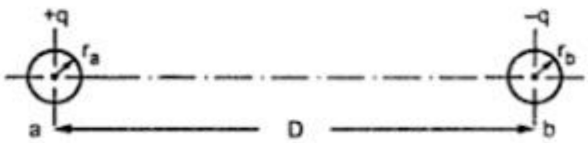
Capacitance of Single Phase Line

Capacitance between the two conductors of a two wire line is the charge on the conductor per unit of potential difference between them. Capacitance of the line per unit length is given by,

$$C = q/v \quad \text{F/m}$$

The capacitance of single phase line is obtained by substituting in above equation v in terms of q.

Consider a single phase overhead transmission line consisting of two conductors 'a' and 'b' which are separated by a distance of D in air as shown in the Fig. 1. The charges on each conductors are respectively +q and -q coulombs per meter length.



The voltage V_{ab} between the two conductors can be obtained by finding the potential difference between the two conductors of the line. Firstly let us find the voltage drop due to charge q on conductor a and then finding voltage drop due to charge -q on conductor b. Then by using principle of superposition the voltage drop between conductor a and b is obtained by adding the voltage drops caused by each charge alone.

The voltage drop V_{ab} is therefore given by,

$$V_{ab} = V_{ab} \text{ due to } +q + V_{ab} \text{ due to } -q$$

$$V_{ab} = \left(\frac{q}{2\pi\varepsilon}\right) ln\frac{D}{r_a} + \left(\frac{-q}{2\pi\varepsilon}\right) ln\left(\frac{r_b}{D}\right)$$

$$= \left(\frac{q}{2\pi\varepsilon}\right)\left[ln\frac{D}{r_a} - ln\frac{r_b}{D}\right]$$

$$V_{ab} = \left(\frac{q}{2\pi\varepsilon}\right) ln\left(\frac{D^2}{r_a r_b}\right) \text{volts}$$

The capacitance between the conductors is given by,

$$C_{ab} = \frac{q}{V_{ab}} = \frac{2\pi\varepsilon}{ln\left(\frac{D^2}{r_a r_b}\right)} \text{ F/m} = \frac{2\pi\varepsilon_o\varepsilon_r}{ln\left(\frac{D^2}{r_a r_b}\right)} \text{ F/m}$$

For air $\varepsilon_r = 1$

$$\therefore \quad C_{ab} = \frac{2\pi\varepsilon_o}{ln\left(\frac{D^2}{r_a r_b}\right)}$$

If we take $r_a = r_b = r$ then

$$C_{ab} = \frac{2\pi\varepsilon_o}{ln\left(D^2/r^2\right)} = \frac{\pi\varepsilon_o}{ln(D/r)} \text{ F/m}$$

If it is required to find the capacitance between one of the conductors and the neutral point then it is given by,

$$C_n = C_{an} = C_{bn} = \frac{2\pi\varepsilon_o}{ln(D/r)}$$

The idea of capacitance to neutral is shown in the Fig. 2.

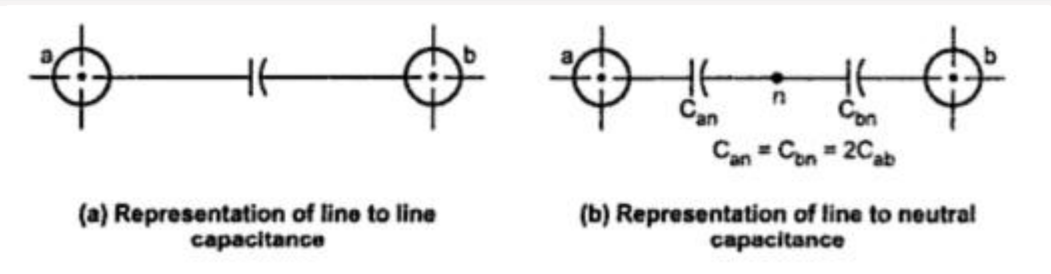


Fig. 2

The equations derived so far related to the capacitance are based on the assumption of uniform charge distribution over the surface of the conductor. In presence of other charges, the distribution will not be uniform and the equations which are derived will not give accurate results. However the nonuniformity of charge distribution can be neglected in case of overhead lines as only 0.010% of error is caused for a close spacing of ratio of D/r = 50.

If instead of a solid round conductor, we have a stranded conductor then the above equation will produce error slightly. The error will be small as only the field very close to the surface of the conductor is not the same as the field at the surface of cylindrical conductor. The outside radius of conductor is used for evaluating capacitance.

If we are having the value of then capacitive reactance can be easily obtained.

Permittivity of free air taken as unity.

Capacitive reactance, $$X_C = \frac{1}{2\pi f C} = \frac{2.862}{f} \times 10^9 \; ln \frac{D}{r} \; \Omega m$$

The above expression gives X_C for 1 m of line since capacitive reactance is in parallel along the line, X_C is divided by the length of line in meters to obtain the value of X_C in ohms to neutral for entire length of line. Standard tables are available for outside diameters of most widely used sizes of ACSR

Effect of Earth on Capacitance of Transmission Line

The capacitance of transmission line is affected by the presence of earth. Because of earth, electric field of a line is reduced. If we assume that the earth is a perfect conductor in the form of a horizontal plane of infinite extent, we realize that the electric field of charged conductors above the earth is not the same as it would be if the equipotential surface of earth were not present.

The method of images is used while considering this type of problems. For this consider a single phase line having 2 conductors as shown in the Fig. 1.

A fictitious conductor is placed below each conductor of the same size and shape as the overhead conductor lying directly below the original conductor at a distance equal to twice the distance of the conductor above the plane of ground. If the earth is removed and a charge equal and opposite to that an overhead conductor is assumed on the fictitious conductor, the plane midway between conductor and its image is an equipotential surface and occupies the same position as the equipotential surface of earth. This fictitious conductor is called image conductor having the charge opposite to that of overhead conductor.

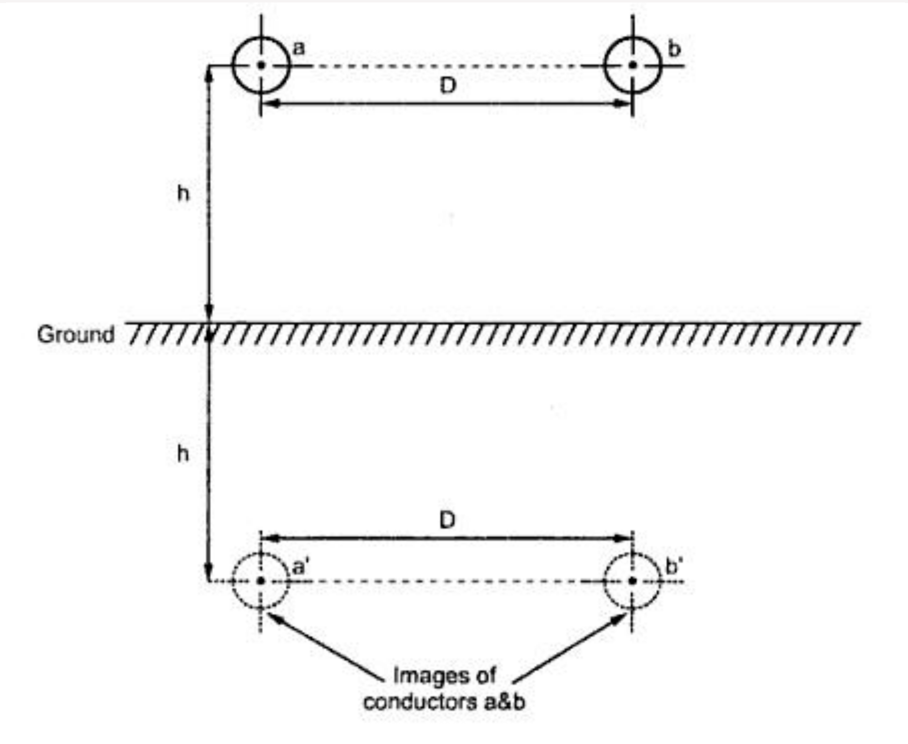


Fig. 1

The voltage V_{ab} is given as,

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left\{q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right\} + \frac{1}{2\pi\varepsilon}\left\{-q_a \ln\frac{\sqrt{D^2+4h^2}}{2h} - q_b \ln\frac{2h}{\sqrt{D^2+4h^2}}\right\}$$

$$= \frac{1}{2\pi\varepsilon}\left\{q_a \ln\frac{2hD}{r\sqrt{D^2+4h^2}} + q_b \ln\frac{r\sqrt{D^2+4h^2}}{2hD}\right\}$$

But $q_b = -q_a$

$$= \frac{1}{2\pi\varepsilon}\left\{q_a\, ln\frac{2hD}{r\sqrt{D^2+4h^2}} - q_a\, ln\frac{r\sqrt{D^2+4h^2}}{2hD}\right\}$$

$$= \frac{1}{2\pi\varepsilon} q_a\, ln\left[\frac{4h^2D^2}{r^2\,(D^2+4h^2)}\right] = \frac{1}{2\pi\varepsilon} q_a\, ln\left[\frac{D^2}{r^2\left(1+\frac{D^2}{4h^2}\right)}\right]$$

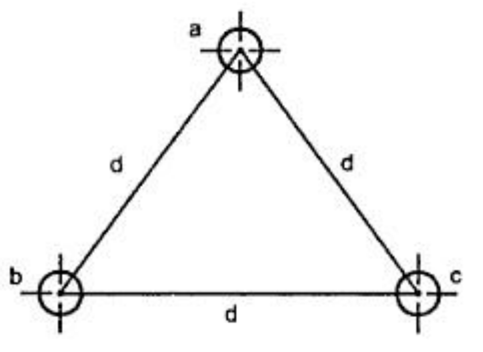
$$= \frac{1}{2\pi\varepsilon} 2q_a\, ln\left[\frac{D}{r\sqrt{r^2\left(1+\frac{D^2}{4h^2}\right)}}\right] = \frac{1}{\pi\varepsilon} q_a\, ln\left[\frac{D}{r\sqrt{1+\frac{D^2}{4h^2}}}\right]$$

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\pi\varepsilon}{ln\left[\frac{D}{r\sqrt{1-\frac{D^2}{4h^2}}}\right]}$$

Comparing above equation with expression for capacitance of single phase line without considering the effect of earth, we can see that earth tries to increase the capacitance of line by small amount. But the effect is negligible if the conductors are high above ground compared to distances between them.

Capacitance of a 3ph Line With Equilateral Spacing

Consider the three conductors a, b and c of 3 phase overhead transmission line having the charges q_a, q_b and q_c respectively as shown in Fig. 1. Let the conductors be separated from each other by a distance of d from each other and placed on the vertices of equilateral triangle.



F ig. 1

The radius of each conductor is say r. The voltage V_{ab} of the three phase line due to only charges on conductors a and b is given by,

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a\, ln\frac{d}{r} + q_b ln\frac{r}{d}\right)$$

Voltage V_{ab} due to only charge q_c is zero as uniform charge distribution over the surface of the conductor is equivalent to a concentrated charge at the centre of conductor.

$$\therefore \quad V_{ab} = \frac{q_c}{2\pi\varepsilon} ln\frac{d}{d}$$

Considering all the three charges in writing the voltage equation we have,

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \, ln\frac{d}{r} + q_b \, ln\frac{r}{d} + q_c \, ln\frac{d}{d}\right] \quad ...(1)$$

Similarly,
$$V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \, ln\frac{d}{r} + q_b \, ln\frac{d}{d} + q_c \, ln\frac{r}{d}\right] \quad ...(2)$$

Adding equations (1) and (2),

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left[2q_a \, ln\frac{d}{r} + (q_b + q_c) ln\frac{r}{d}\right]$$

Adding equations (1) and (2),

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left[2q_a \, ln\frac{d}{r} + (q_b + q_c) ln\frac{r}{d}\right]$$

The voltages are sinusoidal and expressed as the phasors. In absence of other charges in the vicinity the sum of the charges is zero i.e.

$$q_a + q_b + q_c = 0$$

$$\therefore \quad q_b + qc = - q_a$$

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left[2q_a \, ln\frac{d}{r} - q_a \, ln\frac{r}{d}\right] = \frac{1}{2\pi\varepsilon}\left[2q_a ln\frac{d}{r} + q_a ln\frac{d}{r}\right]$$

$$= \frac{3q_a}{2\pi\varepsilon} ln\frac{d}{r}$$

The Fig. 2 shows phasor diagram of balanced voltages of three phase line.

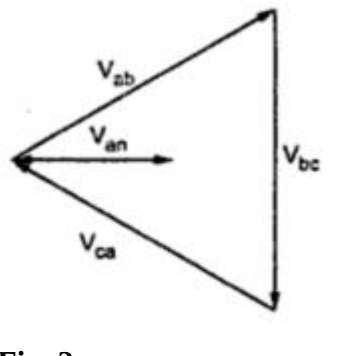


Fig. 2

$$V_{ab} = \sqrt{3}\,V_{an} \angle 30° = \sqrt{3}\,V_{an} \,[0.866 + j\,0.5]$$

$$V_{ac} = -V_{ca} = \sqrt{3}\,V_{an} \angle -30°$$

$$= \sqrt{3}\,V_{an} \,[0.866 - j\,0.5]$$

Adding above equations we get,

$$V_{ab} + V_{ac} = 3\,V_{an}$$

$$3V_{an} = \frac{3q_a}{2\pi\varepsilon} ln\frac{d}{r}$$

$$\therefore\; V_{an} = \frac{q_a}{2\pi\varepsilon} ln\frac{d}{r}$$

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{ln\frac{d}{r}}\;F/m = \frac{2\pi\varepsilon_o\varepsilon_r}{ln\frac{d}{r}}\;F/m$$

For air, $\varepsilon_r = 1$

$$\therefore\qquad C_{an} = 2\pi\varepsilon_o \,/\, ln\frac{d}{r}$$

It can be seen that capacitance to neutral for single phase and equilaterally spaced three phase lines is same.

The current associated with capacitance of a transmission line is termed as charging current. In case of single phase circuits, the charging current of line to line voltage and line to line susceptance.

$$I_C = j\;\omega\, C_{ab}\, V_{ab}$$

In case of three phase circuits, the charging current is found by product of voltage to neutral and capacitive susceptance to neutral. The charging current obtained is for one phase. The current in any phase is given by,

$$I_C = j\;\omega\, C_n\, V_{an}$$

The charging current is not same everywhere as the rms voltage along line varies. For obtaining the charging current the value of voltage used is that for which the line is designed which may not be actual voltage at either generating station or a load.

Capacitance of a 3Ph Line With Unsymmetrical Spacing

The calculation of capacitance in case of conductors in three phase system which are not equally spaced is difficult. If the line is untransposed the capacitances of each phase to neutral is not same.

In case of transposed line the average capacitance of each line to neutral over a complete transposition cycle is same as the average capacitance to neutral of any other phase. Each conductor occupies the same position of every other conductor after equal distance. The effect of unsymmetry between the lines is small and calculations are carried out by considering transposition of lines.

The Fig. 1 shows three phase line with unsymmetrical spacing. The radius of each conductor is r.

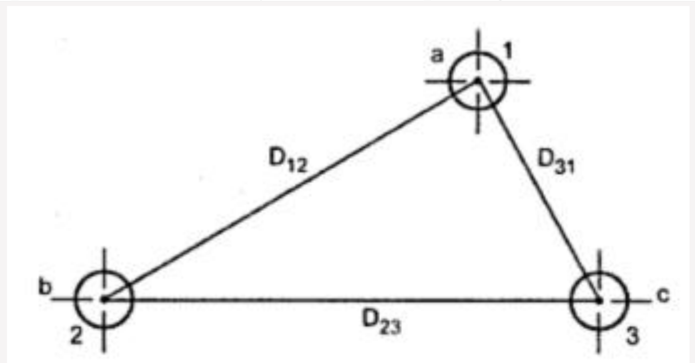


Fig. 1

When phase 'a' is in position 1, 'b' in position 2 and 'c' is in position 3.

$$V_{ab_1} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right)$$

When phase 'a' is in position 2, 'b' in position 3 and 'c' is in position 1.

$$V_{ab_2} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right)$$

When phase 'a' is in position 3, 'b' in position 1 and 'c' is in position 2.

$$V_{ab_3} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right)$$

Average voltage between conductors 'a' and 'b' is given by

$$V_{ab} = \frac{V_{ab_1} + V_{ab_2} + V_{ab_3}}{3} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln\frac{D_{23}D_{31}D_{12}}{D_{31}D_{12}D_{23}}\right)$$

$$= \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}}\right)$$

Let $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

$$= \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{eq}}{r} + q_b \ln\frac{r}{D_{eq}}\right)$$

Similarly average voltage drop between a and c is given by,

$$V_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{eq}}{r} + q_c \ln\frac{r}{D_{eq}}\right)$$

We have, $V_{ab} + V_{ac} = 3\,V_{an}$

$$3V_{an} = \frac{1}{2\pi\varepsilon}\left[2q_a \ln\frac{D_{eq}}{r} + q_b \ln\frac{r}{D_{eq}} + q_c \ln\frac{r}{D_{eq}}\right]$$

For balanced circuit $(q_a + q_b + q_c) = 0$

$$3\,V_{an} = \frac{1}{2\pi\varepsilon}\left[3\,q_a\, ln\frac{D_{eq}}{r}\right]$$

$$V_{an} = \frac{1}{2\pi\varepsilon}\, q_a\, ln\frac{D_{eq}}{r}$$

$$C_{an} = \frac{q_{an}}{V_{an}} = \frac{2\pi\varepsilon}{ln\left(\frac{D_{eq}}{r}\right)} \quad \textbf{F/m}$$

Capacitance of 3Ph Line with more than One Circuit

Consider the arrangement of conductors shown in the Fig. 1.

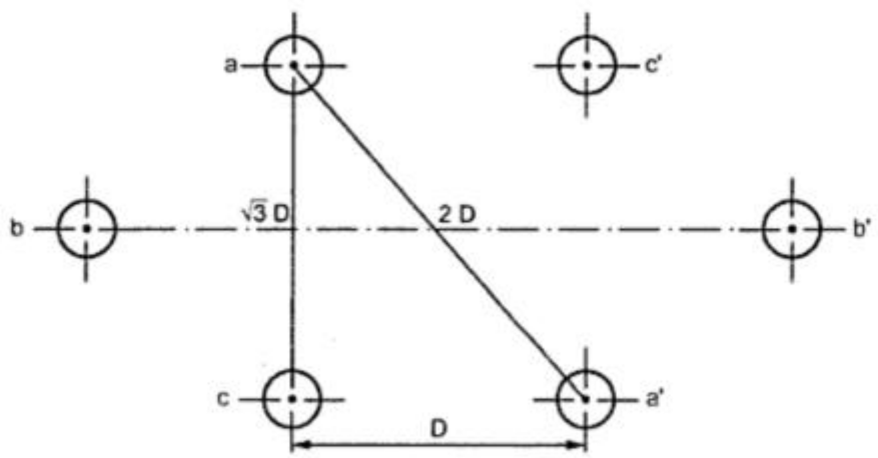


Fig. 1

The voltage between phases a and b is given by,

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left\{\begin{array}{l} q_a\left[ln\left(\frac{D}{r}\right)+ln\left(\frac{\sqrt{3}\,D}{2D}\right)\right]+q_b\left[ln\left(\frac{r}{D}\right)+ln\left(\frac{2D}{\sqrt{3}\,D}\right)\right] \\ +q_c\left[ln\left(\frac{D}{\sqrt{3}\,D}\right)+ln\left(\frac{\sqrt{3}\,D}{D}\right)\right]\end{array}\right\}$$

$$= \frac{1}{2\pi\varepsilon}\left\{q_a\left(ln\frac{\sqrt{3}\,D}{2r}\right)+q_b\left(ln\frac{2r}{\sqrt{3}\,D}\right)+q_c\, ln\left(\frac{\sqrt{3}\,D^2}{\sqrt{3}\,D^2}\right)\right\}$$

$$= \frac{1}{2\pi\varepsilon}\left\{q_a\, ln\frac{\sqrt{3}\,D}{2r}+q_b\, ln\frac{2r}{\sqrt{3}\,D}\right\}$$

Similarly, $$V_{ac} = \frac{1}{2\pi\varepsilon}\left\{q_a\, ln\frac{\sqrt{3}\,D}{2r}+q_c\, ln\frac{2r}{\sqrt{3}\,D}\right\}$$

We have, $V_{ab} + V_{ac} = 3\,V_{an}$

$$3\,V_{an} = \frac{1}{2\pi\varepsilon}\left\{2q_a\, ln\frac{\sqrt{3}D}{2r} + (q_b+q_c)\, ln\frac{2r}{\sqrt{3}D}\right\}$$

$q_a + q_b + q_c = 0$

$\therefore \quad q_b + q_c = -\,q_a$

$$3V_{an} = \frac{1}{2\pi\varepsilon}\left\{2q_a\, \frac{\sqrt{3}D}{2r} - (-q_a)\, ln\frac{\sqrt{3}D}{2r}\right\}$$

$$3\,V_{an} = \frac{1}{2\pi\varepsilon}\cdot 3q_a\, ln\frac{\sqrt{3}D}{2r}$$

$$\therefore \quad V_{an} = \frac{q_a}{2\pi\varepsilon}\, ln\frac{\sqrt{3}D}{2r}$$

$$\therefore \quad C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{ln\left(\frac{\sqrt{3}D}{2r}\right)}$$

Capacitance per phase will be nothing but 2 C_{an}

Capacitance of 3Ph Double Circuit with Unsymmetrical Spacing but Transposed

Consider the arrangement of conductors shown in the Fig. 1. It consists of three phase double circuit. The radius of each conductor r.

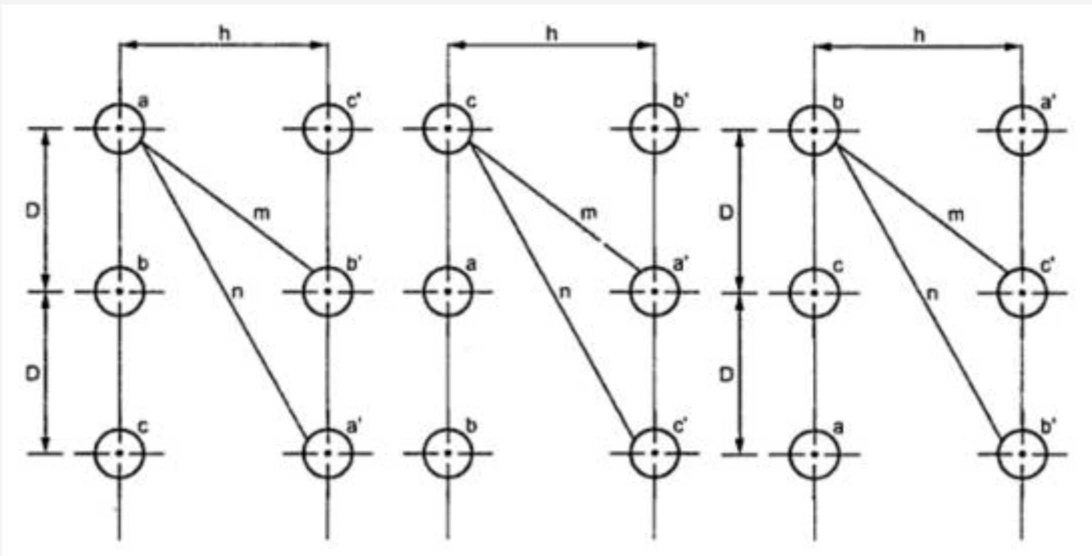


Fig. 1

The voltage between phases a and b can be calculated in order to calculate capacitance.

One complete cycle of transposition is shown in the Fig. 1.

$$V_{ab1} = \frac{1}{2\pi\varepsilon}\left\{q_a\left(ln\frac{D}{r}+ln\frac{m}{n}\right)+q_b\left(ln\frac{r}{D}+ln\frac{h}{m}\right)+q_c\left(ln\frac{D}{2D}+ln\frac{m}{h}\right)\right\}$$

$$V_{ab2} = \frac{1}{2\pi\varepsilon}\left\{q_a\left(ln\frac{D}{r}+ln\frac{m}{h}\right)+q_b\left(ln\frac{r}{D}+ln\frac{h}{m}\right)+q_c\left(ln\frac{2D}{D}+ln\frac{h}{m}\right)\right\}$$

$$V_{ab3} = \frac{1}{2\pi\varepsilon}\left\{q_a\left(ln\frac{2D}{r}+ln\frac{h}{n}\right)+q_b\left(ln\frac{r}{2D}+ln\frac{n}{h}\right)+q_c\left(ln\frac{D}{D}+ln\frac{m}{m}\right)\right\}$$

$$V_{ab} = \frac{1}{3}[V_{ab1}+V_{ab2}+V_{ab3}]$$

$$= \frac{1}{6\pi\varepsilon}\left\{q_a\left(ln\frac{2D^3}{r^3}+ln\frac{m^2h}{n^2h}\right)+q_b\left(ln\frac{r^3}{2D^3}+ln\frac{n^2h}{m^2h}\right)+q_c\left(ln\frac{2D^3}{2D^3}+ln\frac{m^2h}{m^2h}\right)\right\}$$

$$V_{ab} = \frac{1}{6\pi\varepsilon}\left\{q_a\,ln\frac{2D^3m^2}{r^3n^2}+q_b\,ln\frac{r^3n^2}{2D^3m^2}\right\}$$

Similarly,

$$V_{ac} = \frac{1}{6\pi\varepsilon}\left\{q_a\,ln\frac{2D^3m^2}{r^3n^2}+q_c\,ln\frac{r^3n^2}{2D^3m^2}\right\}$$

We have, $V_{ab} + V_{ac} = 3\,V_{an}$

$$\therefore \quad 3V_{an} = \frac{1}{6\pi\varepsilon}\left\{2q_a\,ln\frac{2D^3m^2}{r^3n^2}+(q_b+q_c)\,ln\frac{r^3n^2}{2D^3m^2}\right\}$$

Also, $q_a + q_b + q_c = 0$

$$3V_{an} = \frac{1}{6\pi\varepsilon}\left\{3q_a\,ln\frac{2D^3m^2}{r^3n^2}\right\}$$

$$V_{an} = \frac{1}{6\pi\varepsilon}\left\{q_a\,ln\frac{2D^3m^2}{r^3n^2}\right\}$$

$$C_{an} = \frac{q_a}{V_{an}} = \frac{6\pi\varepsilon}{ln\frac{2D^3m^2}{r^3n^2}}$$

$$\therefore \quad C_{an} = \frac{2\pi\varepsilon}{ln\left\{2^{1/3}\left(\frac{D}{r}\right)\left(\frac{m}{n}\right)^{\frac{2}{3}}\right\}}$$

The capacitance per phase will be 2 C_{an} .

Capacitance of system with Bundled Conductors

The Fig. 1 shows the arrangement if the conductors are bundled one

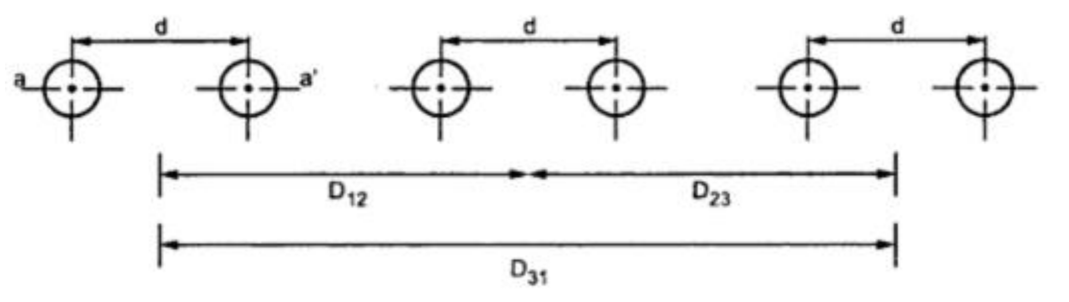


Fig. 1

The conductors of any one bundle are in parallel and charge per bundle is assumed to divide equally between the conductors of bundle.

The composite or stranded conductors touch each other while the bundled conductors are away from each other. The typical distance is about 30 cm and more. The conductors of each phase are connected by using connecting wires at particular length.

Due excessive corona loss, the round conductors are not feasible for use for voltage level more than 230 kV. It is preferable to use hollow conductor in substations while bundled conductors in transmission lines.

Following are advantages of bundled conductors.

1. Low radio interference and corona loss.
2. Reduced voltage gradient at conductor surface.
3. Increase in capacitance.
4. Low reactance due to increase in self GMD.
5. Increase in surge impedance loading.

If the charge on phase a is q_a then charge on each of the conductors a and 'a' will be $q_a/2$. Same is the case with remaining two phases.

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\left(ln\frac{D_{12}}{r} + ln\frac{D_{12}}{d}\right) + \frac{q_b}{2}\left(ln\frac{r}{D_{12}} + ln\frac{d}{D_{12}}\right) + \frac{q_c}{2}\left(ln\frac{D_{23}}{D_{31}} + ln\frac{D_{23}}{D_{31}}\right)\right]$$

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left\{q_a \cdot ln\left[\frac{D_{12}}{\sqrt{r\,d}}\right] + q_b\, ln\cdot\left[\frac{\sqrt{r\,d}}{D_{12}}\right] + q_c\, ln\left[\frac{D_{23}}{D_{31}}\right]\right\}$$

This equation is similar to the expression we have written for 3 phase line with unsymmetrical spacing. Combining the terms we get

$$C_{an} = \frac{2\pi\varepsilon}{ln\left[D_{eq}/\sqrt{rd}\right]} = \frac{2\pi\varepsilon}{ln\left[D_{eq}/D_{sc}^{b}\right]}$$

Here, $D_{sc}^{b} = \sqrt{rd}$ = Modified GMR for capacitor

Thus, for a two strand bundle

$$D_{sc}^{b} = \sqrt[4]{(r \times d)^2}$$
$$= \sqrt{rd}$$

For a three strand bundle

$$D_{sc}^{b} = \sqrt[9]{(r \times d \times d)^3}$$
$$= \sqrt[3]{rd^2}$$

Stranded Conductor

The stranded conductor usually has a central wire which is surrounded by the layers of wires. These layers consists of 6, 12, 18, ...wires successively. Thus the total strands are 7, 13, 19 ....

Such a stranded conductor with 37 strands is shown in the Fig. 2.

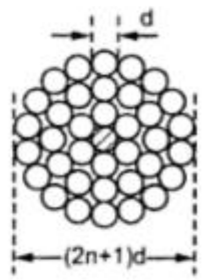


Fig. 2

Let d = diameter of each strand

Then the total diameter of a stranded conductor (cable) is given by,

$$d_c = (2n + 1) d$$

where n = number of layers in which the strands are arranged around central strand.

The stranded conductor is specified as number of strands and diameter of strand.

For example 7/0.295 mm which indicates 7 strands with 0.295 mm diameter of each strand.

If at all the number of layers are not specified then the number of layers can be calculated as number of strands and layers are related to each other by the equation,

$$x = 3n^2 + 3n + 1$$

where x = number of stran n = number of layers

The stranded number of strands in each successive layer from inner to outer is 6, 12, 18, 24

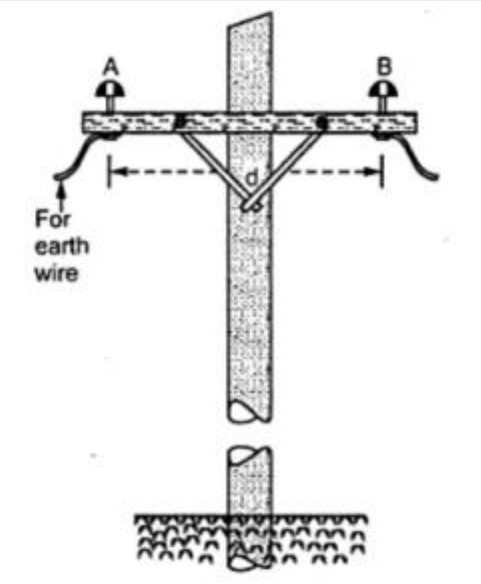
Advantages of Single circuit and double circuit lines.

Methods of Arrangement Conductors

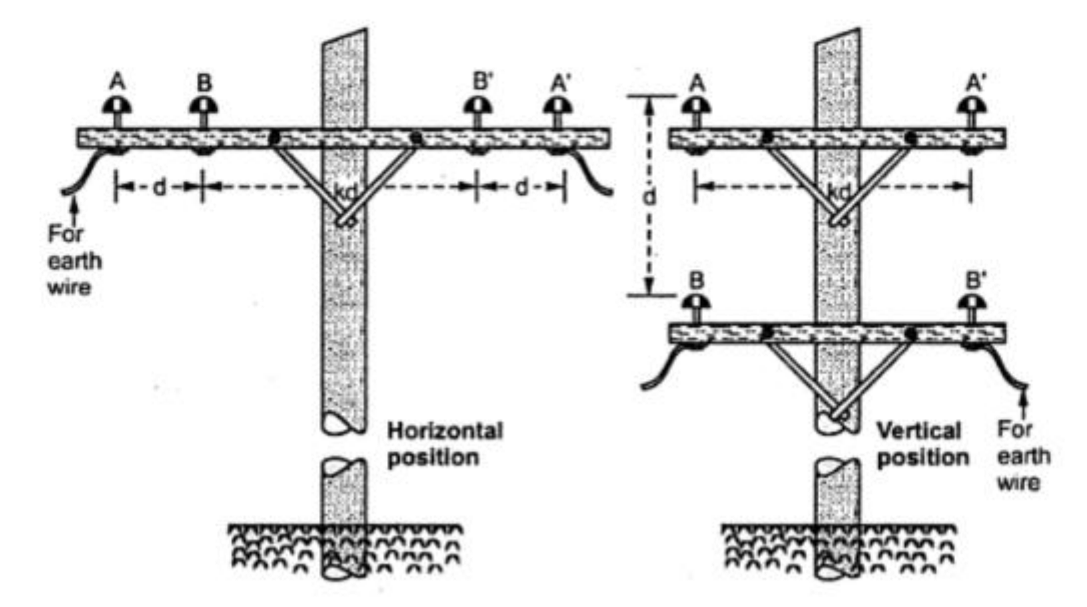
There are various methods of arranging the conductors over the line supports.

i) Single phase circuits :

There are two types of arrangements in single phase circuits viz single circuit and double circuit which are respectively shown in the Fig. 1(a) and (b). In double circuit there are again two subtypes of arrangements viz horizontal and vertical disposition.



(a) Single phase single circuit



(b) Single phase double circuit

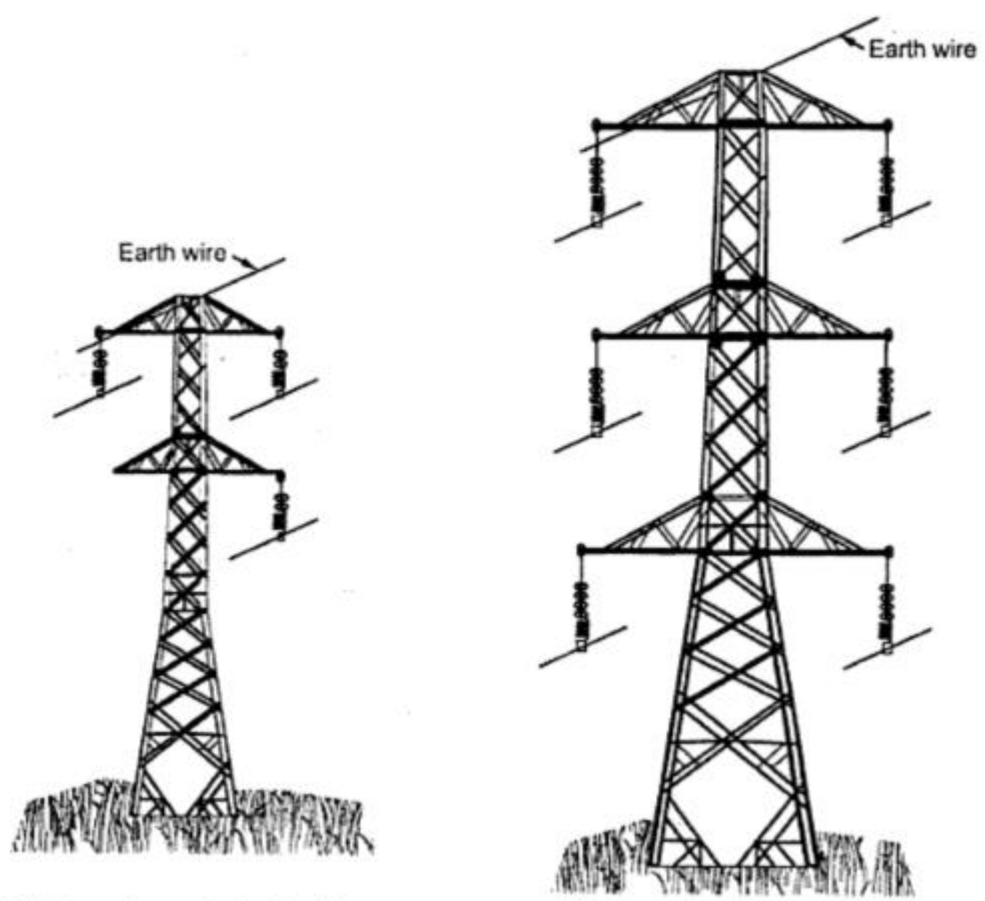
Comparison of Single Circuit and Double Circuit

The comparison of single circuit and double circuit as given below.

Sr. No.	Single circuit arrangement	Double circuit arrangement
1.	This type of arrangement is less dangerous during repair work.	This type of arrangement is comparatively dangerous.
2.	From continuity of supply point of view, the circuit is less reliable.	With reference to continuity of supply, the circuit is much reliable.
3.	It requires lesser foundation and less weight of steel tower member.	It requires more foundation as the structure is of heavier members. The height of tower is more.
4.	The spacing of conductors required is greater.	The spacing of conductors required is lesser.
5.	As spacing of conductors is high the reactance is high.	As spacing of conductors is less, the reactance is also low.

Three phase circuits :

The three phase circuits are also having single circuit or double circuit arrangement. In single circuit arrangement of three phase circuit, the conductors are arranged either at the corners of an equilateral triangle or at the corners of right angled triangle for unequal distance between the conductors. The Fig. 2 shows various types of arrangements of three phase circuits.



(a) Three phase single circuit (b) Three phase double circuit

Question Bank

1. Explain the terms self GMD and mutual GMD and prove that the inductance of a group of parallel wires carrying current can be represented in terms of their geometric distances.
2. Derive an expression for the capacitance of a unsymmetrical spaced but regularly transposed line.
3. Derive the expression for the inductance of a 3 phase transmission line with un symmetrical spacing without transposition. Use the Flux linkage concept.
4. Write short notes on transposition of transmn. Lines.
5. Derive the expression for the inductance of a 3 phase unsymmetrically spaced but transmission line/km.
6. Derive the expression for the capacitance of a 3 phase single ckt. Line with equilateral spacing
7. Show how the inductance of 3 phase transmission . Line with equilateral and symmetrical spacing between conductors can be calculated.
8. Derive the expression for the capacitance of a 3 phase line with unsymmetrical spacing.
9. What is skin effect? Which are the factors influencing skin effect?
10. Derive from first principles, an expression for the inductance per phase per km of a 3 phase regularly transposed trn. Line. The conductors are of diamt. d mt. and placed at the corner of a triangle of sides a, b, c.
11. Calculate the inductance of single phase two wire line starting from fundamentals.

MODULE 3

MODULE-3

Performance of transmission lines

Classification of transmission lines

Short Transmission Lines

The transmission lines are categorized as three types

1) Short transmission line – the line length is up to 80 km

2) Medium transmission line – the line length is between 80km to 160 km

3) Long transmission line – the line length is more than 160 km

$$\text{Efficiency of transmission line} = \frac{\text{power delivered at receiving end}}{\text{power sent from sending end}} \times 100\ \%$$

Whatever may be the category of transmission line, the main aim is to transmit power from one end to another. Like other electrical system, the transmission network also will have some power loss and voltage drop during transmitting power from sending end to receiving end. Hence, performance of transmission line can be determined by its efficiency and voltage regulation.

Power sent from sending end – line losses = power delivered at receiving end

Voltage regulation of transmission line is measure of change of receiving end voltage from no-load to full load condition.

$$\%\ \text{regulation} = \frac{\text{no load receiving end voltage} - \text{full load receiving end voltage}}{\text{full load voltage}} \times 100\ \%$$

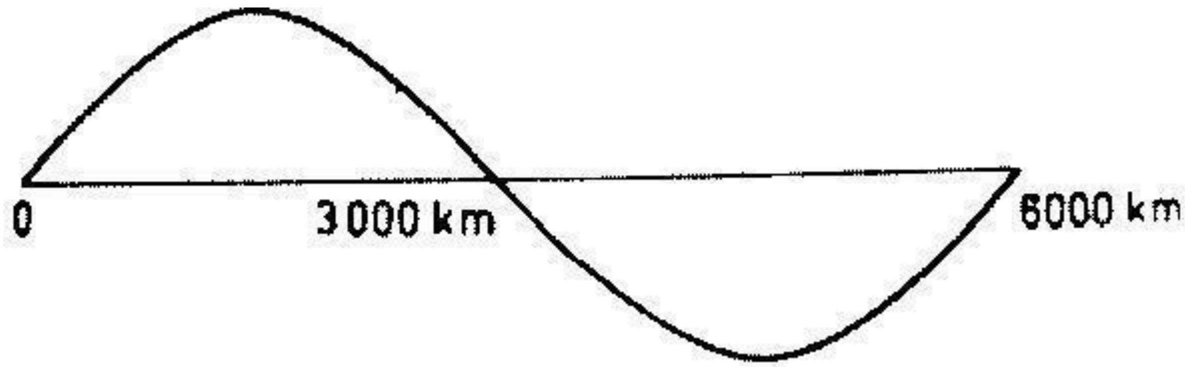
Every transmission line will have three basic electrical parameters. The conductors of the line will have resistance, inductance, and capacitance. As the transmission line is a set of conductors being run from one place to another supported by transmission towers, the parameters are distributed uniformly along the line.

The electrical power is transmitted over a transmission line with a speed of light that is $3X10^8$ m / sec. Frequency of the power is 50Hz. The wave length of the voltage and current of the power can be determined by the equation given below, f.λ = v where f is power frequency, &labda is wave length and v is the speed of light.

$$\text{Therefore, } \lambda = \frac{v}{f}$$

$$\lambda = \frac{3X10^8}{50} = 6X10^6 \text{ meters} = 6000 \text{ km.}$$

Hence the wave length of the transmitting power is quite long compared to the generally used line length of transmission line.



Voltage distribution of 50 Hz supply

For this reason, the transmission line, with length less than 160 km, the parameters are assumed to be lumped and not distributed. Such lines are known as electrically short transmission line. This electrically short transmission lines are again categorized as short transmission line (length up to 80 km) and medium transmission line(length between 80 and 160 km). The capacitive parameter of short transmission line is ignored whereas in case of medium length line the capacitance is assumed to be lumped at the middle of the line or half of the capacitance may be considered to be lumped at each ends of the transmission line. Lines with length more than 160 km, the parameters are considered to be distributed over the line. This is called long transmission line.

ABCD PARAMETERS

A major section of power system engineering deals in the transmission of electrical power from one particular place (eg. Generating station) to another like substations or distribution units with maximum efficiency. So its of substantial importance for power system engineersto be thorough with its mathematical modeling. Thus the entire transmission system can be simplified to a **two port network** for the sake of easier calculations.

The circuit of a 2 port network is shown in the diagram below. As the name suggests, a 2 port network consists of an input port PQ and an output port RS. Each port has 2 terminals to connect itself to the external circuit. Thus it is essentially a 2 port or a 4 terminal circuit, having

Supply end voltage = VS

and Supply end current = IS

Given to the input port P Q.

And there is the Receiving end Voltage = VR

and Receiving end current = IR

Given to the output port R S.

As shown in the diagram below.

Now the **ABCD parameters** or the transmission line parameters provide the link between the supply and receiving end voltages and currents, considering the circuit elements to be linear in nature.

Thus the relation between the sending and receiving end specifications are given using **ABCD parameters** by the equations below.

VS = A VR + B IR ————————-(1)

IS = C VR + D IR ————————-(2)

Now in order to determine the ABCD parameters of transmission line let us impose the required circuit conditions in different cases.

ABCD parameters, when receiving end is open circuited

The receiving end is open circuited meaning receiving end current IR = 0.
Applying this condition to equation (1) we get.

$$V_S = A\,V_R + B\,0 \Rightarrow V_S = A\,V_R + 0$$
$$A = \left.\frac{V_S}{V_R}\right|_{I_R = 0}$$

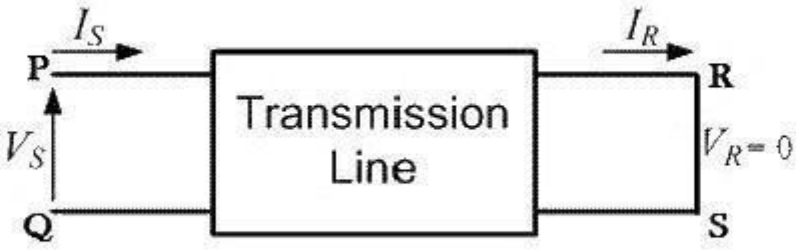
Thus its implies that on applying open circuit condition to ABCD parameters, we get parameter A as the ratio of sending end voltage to the open circuit receiving end voltage. Since dimension wise A is a ratio of voltage to voltage, A is a dimension less parameter.

Applying the same open circuit condition i.e IR = 0 to equation (2)

$$I_S = C\,V_R + D\,0 \Rightarrow I_S = C\,V_R + 0$$
$$C = \left.\frac{I_S}{V_R}\right|_{I_R = 0}$$

Thus its implies that on applying open circuit condition to ABCD parameters of transmission line, we get parameter C as the ratio of sending end current to the open circuit receiving end voltage. Since dimension wise C is a ratio of current to voltage, its unit is mho.
Thus C is the open circuit conductance and is given
by C = IS ⁄ VR mho.

ABCD parameters when receiving end is short circuited



Receiving end is short circuited meaning receiving end voltage VR = 0
Applying this condition to equation (1) we get

$$V_S = A\,0 + B\,I_R \Rightarrow V_S = 0 + B\,I_R$$

$$B = \left.\frac{V_S}{I_R}\right|_{V_R = 0}$$

Thus its implies that on applying short circuit condition to ABCD parameters, we get parameter B as the ratio of sending end voltage to the short circuit receiving end current. Since dimension wise B is a ratio of voltage to current, its unit is Ω. Thus B is the short circuit resistance and is

given by

B = VS ⁄ IR Ω.

Applying the same short circuit condition i.e VR = 0 to equation (2) we get

$$I_S = C\,0 + D\,I_R \Rightarrow I_S = 0 + D\,I_R$$

$$D = \left.\frac{I_S}{I_R}\right|_{V_R = 0}$$

Thus its implies that on applying short circuit condition to ABCD parameters, we get parameter D as the ratio of sending end current to the short circuit receiving end current. Since dimension wise D is a ratio of current to current, it's a dimension less parameter. ∴ the ABCD parameters of transmission line can be tabulated as:-

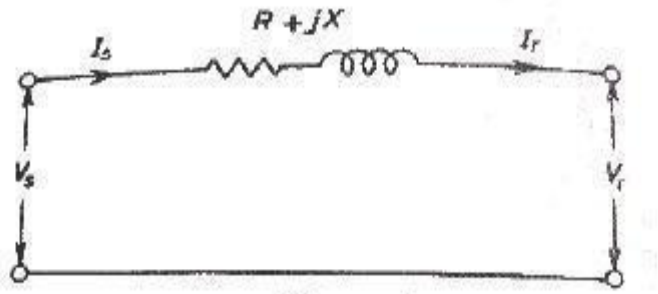
Parameter	Specification	Unit
A = VS / VR	Voltage ratio	Unit less
B = VS / IR	Short circuit resistance	Ω
C = IS / VR	Open circuit conductance	mho
D = IS / IR	Current ratio	Unit less

SHORT TRANSMISSION LINE

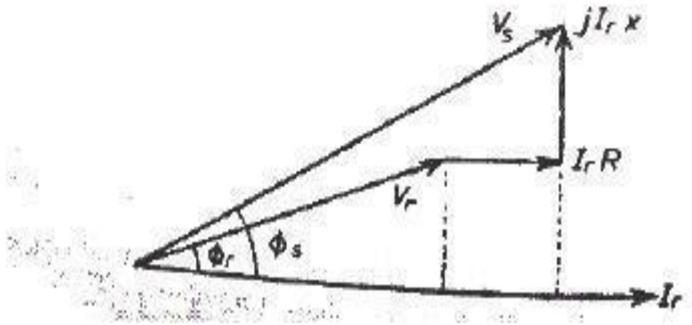
The transmission lines which have length less than 80 km are generally referred as **short transmission lines**.

For short length, the shunt capacitance of this type of line is neglected and other parameters like resistance and inductance of these short lines are lumped, hence the equivalent circuit is represented as given below,

Let's draw the vector diagram for this equivalent circuit, taking receiving end current Ir as reference.The sending end and receiving end voltages make angle with that reference receiving end current, of φs and φr, respectively.



(a)



As the shunt capacitance of the line is neglected, hence sending end current and receiving end current is same, i.e.

Is = Ir.

Now if we observe the vector diagram carefully, we will get, Vs is approximately equal to

Vr + Ir.R.cosφr + Ir.X.sinφr

That means,

Vs ≅ Vr + Ir.R.cosφr + Ir.X.sinφr as the it is assumed that φs ≅ φr

As there is no capacitance, during no load condition the current through the line is considered as zero, hence at no load condition, receiving end voltage is the same as sending end voltage

As per dentition of voltage regulation,

$$\% \text{ regulation} = \frac{V_s - V_r}{V_r} \times 100\ \%$$

$$= \frac{I_r.R.\cos\varphi_r + I_r.X.\sin\varphi_r}{V_r} \times 100\ \%$$

$$\text{per unit regulation} = \frac{I_r.R}{V_r}\cos\varphi_r + \frac{I_r.X}{V_r}\sin\varphi_r = v_r\cos\varphi_r + v_x\sin\varphi_r$$

$$A = \left.\frac{V_s}{V_r}\right|_{I_r = 0}$$

Here, vr and vx are the per unit resistance and reactance of the short transmission line.

Any electrical network generally has two input terminals and two output terminals. If we consider any complex electrical network in a black box, it will have two input terminals and output terminals. This network is called two – port network. Two port model of a network simplifies the network solving technique. Mathematically a two port network can be solved by 2 by 2 matrixes.

A transmission as it is also an electrical network; line can be represented as two port network. Hence two port network of transmission line can be represented as 2 by 2 matrixes. Here the concept of ABCD parameters comes. Voltage and currents of the network can represented as ,

Vs= AVr + BIr…………(1)

Is= CVr + DIr…………(2)

Where A, B, C and D are different constant of the network.

If we put Ir = 0 at equation (1), we get

Hence, A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimension less.

If we put Vr = 0 at equation (1), we get

That indicates it is impedance of the transmission line when the receiving terminals are short circuited. This parameter is referred as transfer impedance.

$$C = \frac{I_s}{V_r}\bigg|_{I_r = 0}$$

C is the current in amperes into the sending end per volt on open circuited receiving end. It has the dimension of admittance.

$$D = \frac{I_s}{I_r}\bigg|_{V_r = 0}$$

D is the current in amperes into the sending end per amp on short circuited receiving end. It is dimensionless.

Now from equivalent circuit, it is found that,

$$V_s = V_r + I_r Z \text{ and } I_s = I_r$$

Comparing these equations with equation 1 and 2 we get,

A = 1, B = Z, C = 0 and D = 1. As we know that the constant A, B, C and D are related for passive network as

$$AD - BC = 1.$$

Here, A = 1, B = Z, C = 0 and D = 1

$$\Rightarrow 1.1 - Z.0 = 1$$

So the values calculated are correct for short transmission line.

From above equation (1),

$$V_s = AV_r + BI_r$$

When Ir = 0 that means receiving end terminals is open circuited and then from the equation 1, we get receiving end voltage at no load

$$V_r' = \frac{V_s}{A}$$

and as per definition of voltage regulation,

$$\% \text{ voltage regulation} = \frac{V_s / A - V_r}{V_r} X100 \%$$

MEDIUM TRANSMISSION LINE

The transmission line having its effective length more than 80 km but less than 250 km, is generally referred to as a **medium transmission line**. Due to the line length being considerably high, admittance Y of the network does play a role in calculating the effective circuit parameters, unlike in the case of short transmission lines. For this reason the modelling of a **medium length transmission line** is done using lumped shunt admittance along with the lumped impedance in series to the circuit.

These lumped parameters of a medium length transmission line can be represented using two different models, namely.

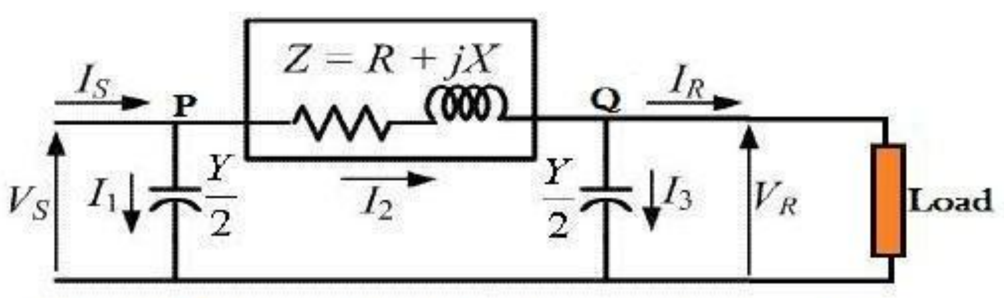
1) Nominal **Π** representation.

2) Nominal **T** representation.

Let's now go into the detailed discussion of these above mentioned models.

Nominal Π representation of a medium transmission line

In case of a nominal **Π** representation, the lumped series impedance is placed at the middle of the circuit where as the shunt admittances are at the ends. As we can see from the diagram of the Π network below, the total lumped shunt admittance is divided into 2 equal halves, and each half with value Y ⁄ 2 is placed at both the sending and the receiving end while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol **Π**, and for this reason it is known as the nominal Π representation of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.



Nominal Π network of medium transmission line.

As we can see here, VS and VR is the supply and receiving end voltages respectively, and Is is the current flowing through the supply end.

IR is the current flowing through the receiving end of the circuit.

I1 and I3 are the values of currents flowing through the admittances.

And I2 is the current through the impedance Z.

Now applying KCL, at node P, we get. IS = I1 + I2 ——————(1)

Similarly applying KCL, to node Q. I2 = I3 + IR ——————(2)

Now substituting equation (2) to equation (1) IS = I1 + I3 + IR

$$= \frac{Y}{2}V_S + \frac{Y}{2}V_R + I_R \text{--------}(3)$$

Now by applying KCL to the circuit,

VS = VR + Z I2

$$= V_R + Z(V_R\frac{Y}{2} + I_R)$$

$$= (Z\frac{Y}{2} + 1)\,V_R + ZI_R \text{ ----------}(4)$$

Now substituting equation (4) to equation (3), we get.

$$I_s = \frac{Y}{2}[(\frac{Y}{2}Z + 1)V_R + ZI_R] + \frac{Y}{2}V_R + I_R$$

$$= Y(\frac{Y}{4}Z + 1)V_R + (\frac{Y}{2}Z + 1)I_R \text{ -----------}(5)$$

Comparing equation (4) and (5) with the standard ABCD parameter equations

VS = A VR + B IR

IS = C VR + D IR

We derive the parameters of a medium transmission line as:

$$A = (\frac{Y}{2}Z + 1)$$

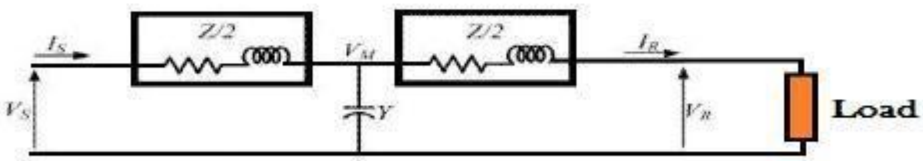
$$B = Z\ \Omega$$

$$C = Y(\frac{Y}{4}Z + 1)$$

$$D = (\frac{Y}{2}Z + 1)$$

Nominal T representation of a medium transmission line

In the **nominal T** model of a medium transmission line the lumped shunt admittance is placed in the middle, while the net series impedance is divided into two equal halves and and placed on either side of the shunt admittance. The circuit so formed resembles the symbol of a capital **T**, and hence is known as the nominal T network of a medium length transmission line and is shown in the diagram below.



Nominal T representation of a medium transmission line.

Here also Vs and Vr is the supply and receiving end voltages respectively, and
Is is the current flowing through the supply end. Ir is the current flowing through the receiving end of the circuit. Let M be a node at the midpoint of the circuit, and the drop at M, be given by
Vm. Applying KVL to the above network we get

$$\frac{V_S - V_M}{Z/2} = Y\,V_M + \frac{V_m - V_R}{Z/2}$$

$$\text{Or } V_M = \frac{2(V_S + V_R)}{YZ + 4} \text{ ---------------(6)}$$

And the receiving end current

$$\text{Or } I_R = \frac{2(V_M - V_R)}{Z/2} \text{ ---------------(7)}$$

Now substituting V_M from equation (6) to (7) we get,

$$\text{Or } I_R = \frac{[(2V_S + V_R)/YZ+4] - V_R}{Z/2}$$

Rearranging the above equation:

$$V_S = (\frac{Y}{2}Z + 1)V_R + Z(\frac{Y}{4}Z + 1)I_R \text{ ---------------(8)}$$

Now the sending end current is

Is = Y VM + IR ——————(9)

Substituting the value of VM to equation (9) we get,

$$\text{Or } I_S = Y\,V_R + (\frac{Y}{2}Z + 1)I_R \text{ ---------------(10)}$$

Again comparing Comparing (8) and (10) with the standard ABCD parameter equations

VS = A VR + B IR

IS = C VR + D IR

The parameters of the **T** network of a medium transmission line are

$$A = (\frac{Y}{2}Z + 1)$$

$$B = Z(\frac{Y}{4}Z + 1)\ \Omega$$

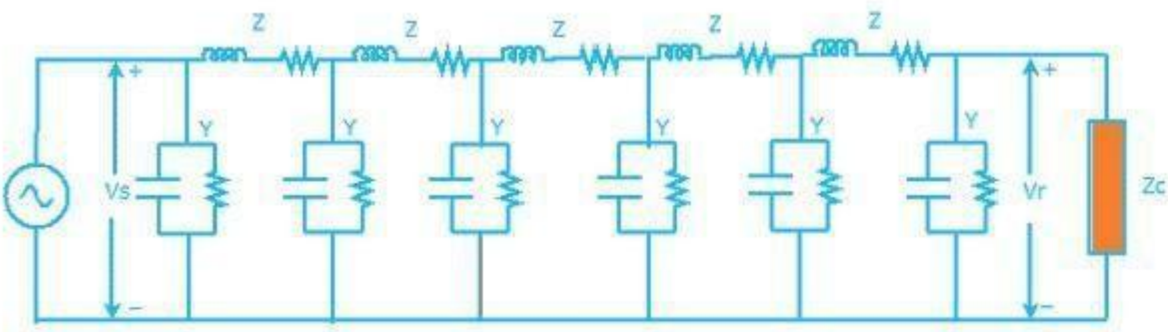
$$C = Y \text{ mho}$$

$$D = (\frac{Y}{2}Z + 1)$$

Performance of Long TransmissionLines

LONG TRANSMISSION LINE

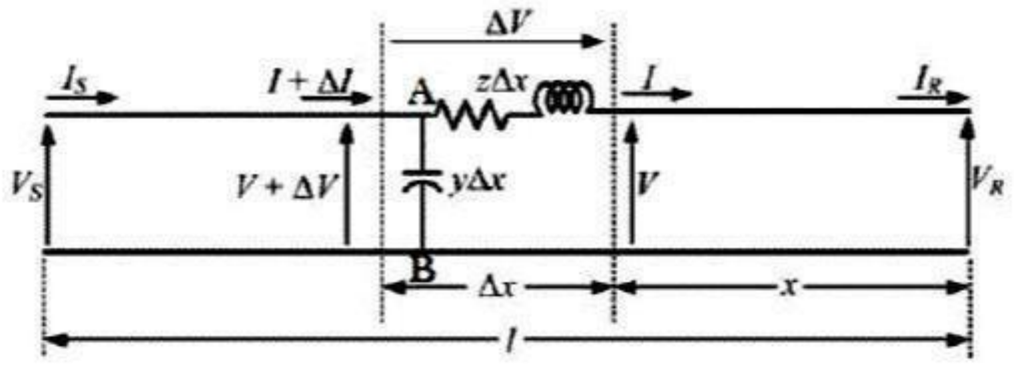
A power transmission line with its effective length of around 250 Kms or above is referred to as a **long transmission line**. Calculations related to circuit parameters (ABCD parameters) of such a power transmission is not that simple, as was the case for a short or medium transmission line. The reason being that, the effective circuit length in this case is much higher than what it was for the former models(long and medium line) and, thus ruling out the approximations considered there like.



Long Transmission Line model

a) Ignoring the shunt admittance of the network, like in a small transmission line model.

b) Considering the circuit impedance and admittance to be lumped and concentrated at a point as was the case for the medium line model.



Long Transmission Line.

Here a line of length l > 250km is supplied with a sending end voltage and current of VS and IS respectively, where as the VR and IR are the values of voltage and current obtained from the receiving end. Lets us now consider an element of infinitely small length Δx at a distance x from the receiving end as shown in the figure where.

V = value of voltage just before entering the element Δx.

I = value of current just before entering the element Δx.

V+ΔV = voltage leaving the element Δx.

I+ΔI = current leaving the element Δx.

ΔV = voltage drop across element Δx.

zΔx = series impedence of element Δx

yΔx = shunt admittance of element Δx

Where Z = z l and Y = y l are the values of total impedance and admittance of the long transmission line.

∴ the voltage drop across the infinitely small element Δx is given by

$\Delta V = I\, z\, \Delta x$

Or $I z = \Delta V / \Delta x$

Or $I z = dV / dx$ ——————————(1)

Now to determine the current ΔI, we apply KCL to node A.

$\Delta I = (V+\Delta V) y\Delta x = V\, y\Delta x + \Delta V\, y\Delta x$

Since the term ΔV yΔx is the product of 2 infinitely small values, we can ignore it for the sake of easier calculation.

∴ we can write $dI / dx = V\, y$ ————————(2)

Now derevating both sides of eq (1) w.r.t x,

$d^2 V / d x^2 = z\, dI / dx$

Now substituting $dI / dx = V y$ from equation (2)

$d^2 V / d x^2 = zyV$

or $d^2 V / d x^2 - zyV = 0$ ————(3)

The solution of the above second order differential equation is given by.

$V = A1\, e^{x\sqrt{yz}} + A2\, e^{-x\sqrt{yz}}$ —————(4)

Derivating equation (4) w.r.to x.

$dV/dx = \sqrt{(yz)}\, A1\, e^{x\sqrt{yz}} - \sqrt{(yz)} A2\, e^{-x\sqrt{yz}}$ ————

(5) Now comparing equation (1) with equation (5)

$$I = \frac{dV}{dx} = \frac{zA_1 e^{x\sqrt{(yz)}}}{\sqrt{(z / y)}} - \frac{zA_1 e^{-x\sqrt{(yz)}}}{\sqrt{(z / y)}} \text{----------------(6)}$$

Now to go further let us define the characteristic impedance Zc and propagation constant δ of a long transmission line as

$Zc = \sqrt{(z/y)}$

Ω $\delta = \sqrt{(yz)}$

Then the voltage and current equation can be expressed in terms of characteristic impedance and propagation constant as

$V = A1\, e^{\delta x} + A2\, e^{-\delta x}$ ————(7)

$I = A1/ Zc\, e^{\delta x} + A2 / Zc\, e^{-\delta x}$ —————(8)

Now at x=0, V= VR and I= Ir. Substituting these conditions to equation (7) and (8) respectively. $VR = A1 + A2$ —————(9)

$IR = A1/ Zc + A2 / Zc$ —————(10)

Solving equation (9) and (10), We get values of A1 and A2 as,

$A1 = (VR + ZCIR) / 2$ And

$A1 = (VR - ZCIR)$

Now applying another extreme condition at x=l, we have V = VS and I = IS.

Now to determine VS and IS we substitute x by l and put the values of A1 and A2 in equation (7) and (8) we get

$VS = (VR + ZC\, IR)e^{\delta l} / 2 + (VR - ZC\, IR)e^{-\delta l}/2$ —————(11)

$IS = (VR / ZC + IR)e^{\delta l}/2 - (VR / ZC - IR)e^{-\delta l}/2$————— (12

By trigonometric and exponential operators we know

$\sinh \delta l = (e^{\delta l} - e^{-\delta l})/2$

And $\cosh \delta l = (e^{\delta l} + e^{-\delta l})/2$

∴ equation(11) and (12) can be re-written as

$V_S = V_R \cosh \delta l + Z_C I_R \sinh \delta l$

$I_S = (V_R \sinh \delta l)/Z_C + I_R \cosh \delta l$

Thus comparing with the general circuit parameters equation, we get the ABCD parameters of a long transmission line as,

$C = \sinh \delta l / Z_C$ $A = \cosh \delta l$ $D = \cosh \delta l$ $B = Z_C \sinh \delta l$

Question Bank

1. Derive expressions for generalized ABCD constants for a long transmission line using rigorous method of analysis.
2. Derive expressions for ABCD constants for a medium transmission line using nominal T model. Hence prove AD-BC = 1.
3. Write short note on classification of transmission lines.
4. Write short note on Ferranti effect.
5. What are ABCD constants of a trasmn. Line? Determine the same for a a medium transmission line using nominal TI model. Hence prove AD-BC =1.
6. Write short note on surge impedance loading.
7. Derive expressions for generalized ABCD constants for equivalent T representation of long transmission line.
8. Discuss the terms voltage regulation and transmission efficiency as applied to transmission line
9. Write and explain the classification of overhead trasmn. Lines.
10. Derive an expression for the efficiency and voltage regulation for short trn. Line giving the vector diagram.

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MODULE 4

MODULE-4

CORONA

Electric-power transmission practically deals in the bulk transfer of electrical energy, from generating stations situated many kilometers away from the main consumption centers or the cities. For this reason the long distance transmission cables are of utmost necessity for effective power transfer, which in-evidently results in huge losses across the system. Minimizing those has been a major challenge for power engineers of late and to do that one should have a clear understanding of the type and nature of losses. One of them being the **corona effect in power system**, which has a predominant role in reducing the efficiency of EHV(extra high voltage lines) which we are going to concentrate on, in this article.

For corona effect to occur effectively, two factors here are of prime importance as mentioned below:-

1) Alternating potential difference must be supplied across the line.

2) The spacing of the conductors, must be large enough compared to the line diameter.

Corona Effect in Transmission Line

When an alternating current is made to flow across two conductors of the transmission line whose spacing is large compared to their diameters, then air surrounding the conductors (composed of ions) is subjected to di-electric stress. At low values of supply end voltage, nothing really occurs as the stress is too less to ionize the air outside. But when the potential difference is made to increase beyond some threshold value of around 30 kV known as the critical disruptive voltage, then the field strength increases and then the air surrounding it experiences stress high enough to be dissociated into ions making the atmosphere conducting. This results in electric discharge around the conductors due to the flow of these ions, giving rise to a faint luminescent glow, along with the hissing sound accompanied by the liberation of ozone, which is readily identified due to its characteristic odor. This phenomena of electrical discharge occurring in transmission line for high values of voltage is known as the **corona effect in power system**. If the voltage across the lines is still increased the glow becomes more and more intense along with hissing noise, inducing very high power loss into the system which must be accounted for.

Factors Affecting Corona

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends :

(i) *Atmosphere.* As corona is formed due to ionsiation of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather

(ii) Conductor size. The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness ofthe surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

(iii) Spacing between conductors. If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(iv) Line voltage. The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

The phenomenon of corona plays an important role in the design of an overhead transmission line. Therefore, it is profitable to consider the following terms much used in the analysis of corona effects:

(*i*) Critical disruptive voltage. *It is the minimum phase-neutral voltage at which corona occurs.* Consider two conductors of radii *r* cm and spaced *d* cm apart. If *V* is the phase-

$$g = \frac{V}{r \log_e \frac{d}{r}} \text{ volts / cm}$$

neutral potential, then potential gradient at the conductor surface is given by:

In order that corona is formed, the value of *g* must be made equal to the breakdown strength of air. The breakdown strength of air at 76 cm pressure and temperature of 25ºC is 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*) and is denoted by *go*. If *Vc* is the phase-neutral potential required under these conditions, then,

$$g_o = \frac{V_c}{r \log_e \frac{d}{r}}$$

where g_o = breakdown strength of air at 76 cm of mercury and 25°C = 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*)

∴ Critical disruptive voltage, $V_c = g_o \, r \log_e \frac{d}{r}$

.

$$\delta = \text{air density factor} = \frac{3\cdot 92b}{273+t}$$

Under standard conditions, the value of $\delta = 1$.

$\therefore$ Critical disruptive voltage, $V_c = g_o\,\delta\,r\log_e\frac{d}{r}$

Correction must also be made for the surface condition of the conductor. This is accounted for by multiplying the above expression by irregularity factor m_o.

$\therefore$ Critical disruptive voltage, $V_c = m_o\,g_o\,\delta\,r\log_e\frac{d}{r}$ kV/phase

where m_o = 1 for polished conductors
= 0·98 to 0·92 for dirty conductors
= 0·87 to 0·8 for stranded conductors

(ii) Visual critical voltage. *It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.*

It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage V_c but at a higher voltage V_v, called *visual critical voltage.* The phase-neutral effective value of visual critical voltage is given by the following empirical formula :

$$V_v = m_v\,g_o\,\delta\,r\left(1+\frac{0\cdot 3}{\sqrt{\delta\,r}}\right)\log_e\frac{d}{r}\ \text{kV/phase}$$

where m_v is another irregularity factor having a value of 1·0 for polished conductors and 0·72 to 0·82 for rough conductors.

(iii) Power loss due to corona. Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by :

$$P = 242\cdot 2\left(\frac{f+25}{\delta}\right)\sqrt{\frac{r}{d}}\,(V-V_c)^2\times 10^{-5}\ \text{kW / km / phase}$$

where f = supply frequency in Hz
V = phase-neutral voltage (*r.m.s.*)
V_c = disruptive voltage (*r.m.s.*) per phase

Advantages and Disadvantages of Corona

Corona has many advantages and disadvantages. In the correct design of a high voltage overhead line, a balance should be struck between the advantages and disadvantages.

Advantages

(i) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(ii) Corona reduces the effects of transients produced by surges.

Disadvantages

(i) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(ii) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidalvoltage drop occurs in the line. This may cause inductive interference with neighbouring communication lines.

Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the sub-stations or bus- bars rated for 33 kV and higher voltages otherwise highly ionised air may cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods :

(i) ***By increasing conductor size***. By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that *ACSR* conductors which have a larger cross-sectional area are used in transmission lines.

(ii) ***By increasing conductor spacing***. By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (*e.g.,* bigger cross arms and supports) may increase to a considerable extent.

Underground Cable

Introduction

The transmission and distribution of an electrical power can be with the help of overhead transmission lines or by underground cables. It has been mentioned that in thickly populated are like towns and cities, the use of overhead lines is not practicable. In such cases electrical energy is transmitted and distributed with the help of underground cables. In its basic form, an underground cable is a conductor provided with proper insulation. As the voltage level increases, the cost of the insulation increases rapidly and thus the use of underground cables is restricted to low and medium voltage distribution.

Comparison of Underground cables and Overhead lines

Compared to overhead lines, the underground cables have the following advantages,

1. It ensures non-interrupted continuity of supply. The possible supply interruptions due to lighting, storms and other weather conditions are eliminated because of underground cables.
2. It requires less maintenance.
3. The accidents caused due to breakage of overhead line conductors are eliminated due to use of underground cables.
4. The voltage drop in the underground cables is less.
5. The life of underground cables is long compared to overhead lines.

6- The beauty of cities and town get maintained due to underground network of cables.

7- The overhead lines use bars conductors which is unsafe in thickly populated area. Hence from safety point of view, the underground cables are more advantageous.

The only drawbacks of underground cables are the extremely high initial cost and insulation problems at high voltages. In India, the big cities have adopted the system of underground cables for the transmission and distribution.

Thus the use of underground cables is mainly for the distribution of an electrical power at low and medium voltages. Its use is almost compulsory at the location where use of overhead lines is not practicable due to the safety reasons such as congested urban area, crossing of wide roads, near gas plants and refineries, near substation etc.

Still the overhead lines also have some advantages compared to the underground cables which are,

1- Long distance transmission is possibly by the overhead lines.

2- The conductors in overhead lines is less expensive.

3- The size of the conductor in overhead lines is less than underground cables due to good heat dissipation in overhead lines.

4- The insulation cost is very less as the air itself acts as an insulation between the conductors. The gas or oil is not required for overhead lines. For high voltage levels, spacing in air can be easily adjusted in case of overhead lines to obtain proper insulation.

5- The erection cost is much less for the overhead lines. The underground cable laying is difficult and complicated.

Depending upon the situation and the requirements, the underground cables or overhead transmission system can be used.

Requirements of the Cables

An underground cable can be defined as the group of individually insulated one or more conductors which is put together and finally provided with number of layers of insulations to give proper mechanical support.

The conductors used in the cables are usually aluminium or annealed copper while the insulation is commonly PVC or other chemical compositions. Many types of cables are available depending upon the nature of conductors, number of conductors, types of insulation used etc. The basic necessary requirements of the cables are,

1- The size of the conductors used must be such that it should be carry the specified load without overheating and keeping the voltage drop well within the permissible limits.

2- As the voltage level for which cables are designed, the insulation thickness must be proper so as to provide high degree of safety and the reliability.

3- The cables must be surrounded by number of layers of an additional insulation so as to give proper mechanical strength and protection. Thus the cables can withstand the rough use at the time of laying them.

4- The material used in the manufacturing of cables must be such that there is complete chemical and physical stability throughout.

Types of Cables

Belted cables

As mentioned earlier, these are used for the voltage levels upto 11 KV. The construction of belted cable is shown in the Fig.1.

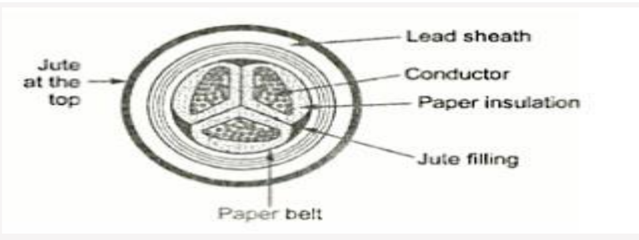


Fig. 1 Belted 3 core cable

The cores are not circular in shape. The core are insulated from each other by use of impregnated paper. The three cores are grouped together and belted with the help of a paper belt. The gaps are filled with fibrous material like jute. This gives circular cross-sectional shape to the cable. The belt is covered with lead sheath which protects cable from moisture and also gives mechanical strength. The lead sheath is finally covered by jute like fibrous compounded material.

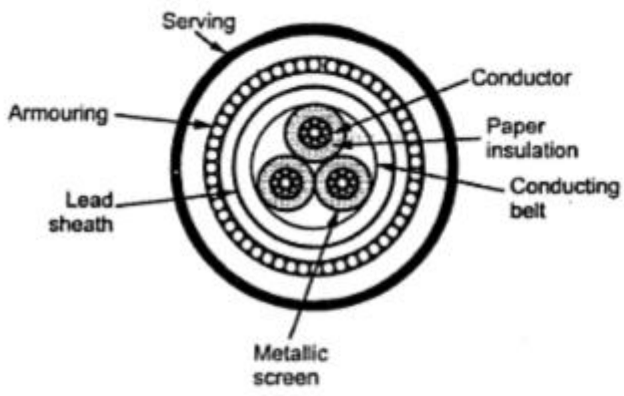
The electrical field in single core cable is radial while it is tangential in case of three core cables. Hence the insulation is subjected to tangential electrical stresses rather than radial one. The paper has good radial strength but not tangential strength. Similarly paper resistance along the radius is much larger than resistance along tangential path. The same is true for dielectric strength also. The fibrous material is also subjected to the tangential electrical stresses, for which, the material is weak. Hence under high voltage cases, the cumulative effect of tangential electrical stresses is to from space inside the cable due to leakage currents. Such air spaces formed inside the insulation is called void formation. This void formation is dangerous because under high voltage, spaces are ionized which deteriorates the insulation which may lead to the breakdown of the insulation. Hence the belted cables are not used for the high voltage levels. Another disadvantage of the belted cable is large diameter of paper belt. Due to this, wrinkles are formed and gaps may be developed if the cable is bended. To overcome all theses difficulties, the screened type cables are used.

Screened Type Cables

These are used for the voltage levels of 22 kV and 33 kV. The two types of screened cables are 1. H type cables and 2. S.L. type cables.

1.H-Type Cables

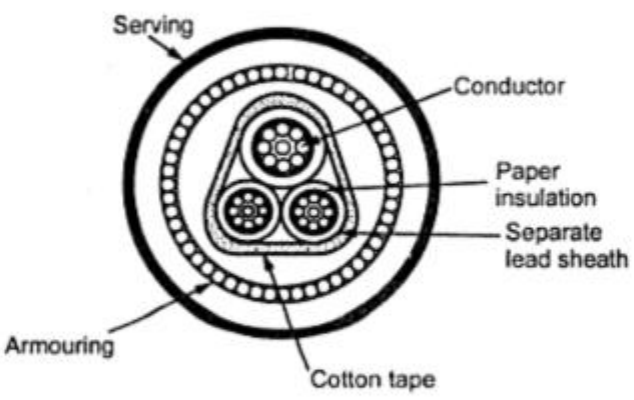
The cable is designed by M.Hochstadter and hence the name given to it is H-Type cable. There is no paper belt in this type of cable. Each conductor in this cable is insulated with a paper, covered with a metallic screen which is generally an aluminium foil. The construction is shown in the Fig. 1. The metallic screen touches each other. Instead of paper belt, the three cores are wrapped with a conducting belt which is usually copper woven fabric tape. Then there is lead sheath. The conducting belt is in electrical contact with the metallic screen and lead sheath. After lead sheath there are layers of bedding, armoring and serving. The metallic screen helps to completely impregnate the cable which avoids the possibility of formation of voids and spaces. The conducting belt, the three metallic screens and lead sheath are at earth potential, due to which electrical stresses are radial in nature. This keeps the dielectric losses to minimum. Another advantage of metallic screens is increase in the heat dissipation which reduces the sheath losses. Due to these advantages, current carrying capacity of these cables increase. In special cases, the use of these cables cable extended upto the 66 kV level.



H-type cable

2 S.L. Cables

The name S.L. stands for separate lead screened cables. In this cable, each core is insulated with impregnated paper and each one is then covered by separate lead sheath. Then there is a cotton tape covering the three cores together using a paper filler material. Then there are the layers of armouring and serving. the difference between H-type and S.L. type cable is that in S.L. type a common lead sheath covering all the three cores is absent while each core is provided with separate lead sheath. This allows bending of the cables as per the requirement. The construction of S.L. type cable is shown in the Fig. 2.



S.L. type cable

The three cores in this type of cable are as good as three separate single core cables.

The various advantages of S.L. type cable are,

1. Due to lead individual lead sheath, core to core fault possibility gets minimised.
2. The electrical stresses are radial in nature.
3. Due to absence of overall lead sheath, bending of cable is easy.
4. The dielectric which gets subjected to electric stresses is paper which is homogeneous hence there is no possibility of formation of voids.
5. Metal sheath increases the heat dissipation which increases the current carrying capacity.

A combination of H-type and S.L. type cable called H.S.L cable also can be used.

The lamination of screened cables which are also called solid type cables are,

1. It uses solid insulation only like paper. When the conductor temperature increases, the paper gets expanded. This eventually stretches the lead sheath.
2. When the load on the cable decreases, it cools down and there is contraction of lead sheath. Due to this air may be drawn into the cable forming voids. This deteriorates the cable insulation.
3. Moisture may be drawn in alongwith the air which deteriorates the dielectric strength of dielectric.
4. Mechanical shock can cause voids. The breakdown strength of voids in much less than insulation. Hence voids can cause permanent damage to the cables.

Super Tension (S.T.) Cables

In solid type cables separate arrangement for avoiding void formation and increasing dielectric strength is not provided. Hence those cables are used maximum upto 66 kV level. The S.T. cables are intended for 132 kV to 275 kV voltage levels.

In such cables, the following methods are specially used to eliminate the possibility of void formation :

1. Instead of solid type insulation, low viscosity oils under pressure is used for impregnation. The channels are used for oil circulation and oil is always kept under pressure. The pressure eliminates completely, the formation of voids.
2. Using inert gas at high pressure in between the lead sheath and dielectric.

Such cables using oil or gas under pressure are called pressure cables and are of two types,

a. Oil filled cables b. Gas Pressure cables

Oil filled Cables

n case of oil filled cables, the channels or ducts are provided within or adjacent to the cores, through which oil under pressure is circulated.

The Fig. 1 shows the construction of single core oil filed cable. It consists of concentric stranded conductors but built around a hollow cylindrical steel spiral core. This hollow core acts as a channel for the oil. The oil channel is filled in a factory and the pressure is maintained in the oil by connecting the oil channel to the tanks which are placed at the suitable distances along the path of the cable.

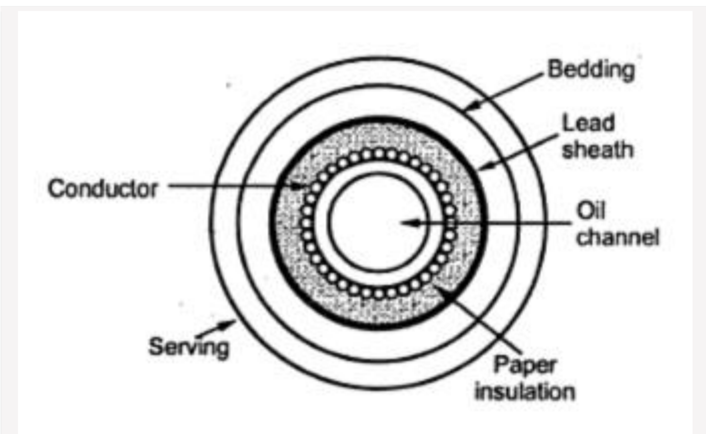


Fig. 1 Conductor channel single core oil filled cable

The oil pressure compresses the paper insulation, eliminating the possibility of formation of voids. When cable is heated, the oil expands but expanded oil is collected in the tank. While when cable is cooled, extra oil is supplied by the tank to maintain the oil pressure. In this type of cable the oil channel is within the conductor, hence it is called single core conductor channel oil filled cables. The other construction of the cable is similar to that of solid type cables.

Another type of single core oil filled cable is the sheath channel oil filled cable. In this type, the conductor is solid with a paper insulation. While the oil ducts are provided between the dielectric and the lead sheath.

The construction of sheath channel oil filled cable is shown in the Fig. 2. The laying of such cables must be done very carefully.

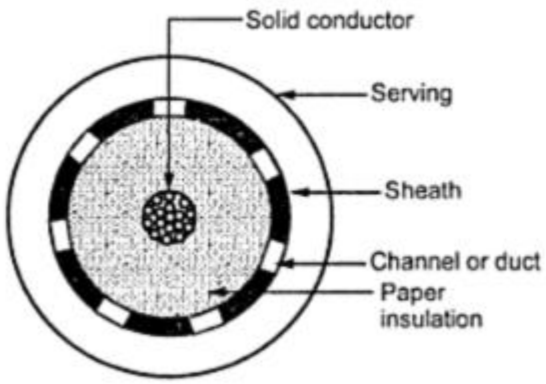


Fig. 2 Sheath channel single core oil filled cable

The three core oil filled cables use the shielded type construction. The oil channels are located in the spaces which are normally occupied by the filler material. The three oil channels are of preforated metal ribbon tubing. All the channels are at earth potential. The construction is shown in the Fig. 3. As the pressure tanks are required all all along the route of these cables, the lengths of these cables are limited. Leakage of oil is another serious problem associated with these cables. Automatic signalling units are located to indicate the fall in oil pressure in any of the phases.

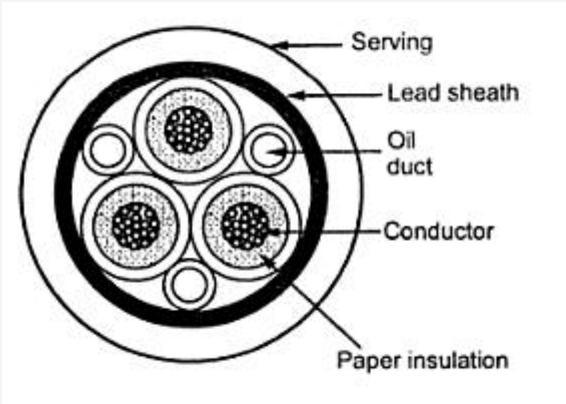


Fig. 3 Oil filled three core cable

Advantages

The various advantages of oil filled cables are,

1. The thickness of insulation required is less hence smaller in size and weight.
2. The thermal resistance is less hence current carrying capacity is more.
3. The possibility of voids is completely eliminated.
4. The allowable temperature range is more than solid type cables.
5. Reduced possibility of earth fault. This is because in case of any defect in lead sheath, oil leakage starts, which can be noticed before earth fault occurs.
6. Perfect impregnation is possible.

Disadvantages

The disadvantages of oil filled cables are,

1. The initial cost is very high.
2. The long length are not possible.
3. The oil leakage is serious problem hence automatic signalling equipment is necessary.
4. The laying of cable is difficult and must be done very carefully.
5. Maintenance of the cables is also complicated.

Gas Pressure Cables

In case of as pressure cables, an inert as like nitrogen at high pressure is introduced. lead sheath and dielectric. The pressure is about 12 to 15 atmospheres. Due to such a high pressure there is a radial compression due to which the ionization is totally eliminated. The working power factors of such cables is also high.

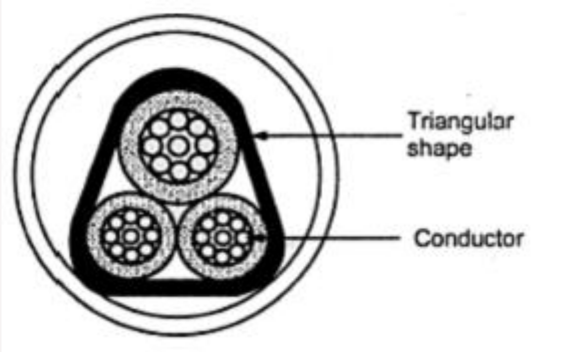


Fig. 1 Gas pressure cables

The Fig. 1 shows the section of a gas pressure cable. The cable id triangular in shape and installed in the steel pipe. The pipe is filled with the nitrogen at 12 to 15 atmospheric pressure. The remaining construction is similar to that of solid type cable but the thickness of lead sheath is 75% of that of solid type cable. There is no bedding and serving. The pressure cable was firstly designed by Hochstadter, Vogel and Bownden.

The triangle shape lead sheath acts as a pressure membrane. The shape reduces the weight and provides the low thermal resistance. The high pressure creates the radial compression to close any voids. The steel pipe is coated with a point to avoid corrosion.

During heating, the cable compound expands and a sheath with acts as a membrane becomes circular in such a case. When cable cools down the gas pressure acting via sheath forces compound to come back to the noncircular normal shape. Due to good thermal characteristics, fire quenching property and high dielectric strength, the gas SF6 is also used in such cables.

Advantages

The various advantages of gas pressure cables are,

1. Gas pressure cables can carry 1.5 times the normal load current and can withstand double the voltage. Hence such cables can be used for ultra high voltage (UHV) levels.
2. Maintenance cost is small.
3. The nitrogen in the steel tube, helps in quenching any fire or flame.
4. No reservoir or tanks required.
5. The power factor is improved.
6. The steel tubes used make the cable laying easy.
7. The ionization and possibility of voids is completely eliminated.

The only disadvantage of this type of cables is very high initial cost.

Insulating Materials for Cables

Number of layers of the various materials is used around the actual conductor in a cable. To isolate the conductor from the surroundings, the conductor is provided with an insulation around it. The materials like paper, vulcanized rubber, PVC etc. are used for providing such an insulation.

The material to be used as an insulation must have the following properties,

1. To prevent leakage current, its insulation resistance must be very high.
2. To avoid electrical breakdown, its dielectric strength must very high.
3. To withstand the mechanical injures, it must be mechanically very strong.
4. It should be flexible.
5. It should be non-hygroscopic so that it will not absorb the moisture from the surroundings.
6. It should be non-inflammable.
7. It should be unaffected by acids and alkalies.
8. It should be capable of withstanding high breakdown voltages.
9. It should have high temperature withstanding capability.

Practically it is not possible to have all these properties in a single material. Hence insulation material is selected depending upon the use of the cable and the quality of insulation required for it. Some changes are done at the time of design depending upon the nature of material selected. For example if the material is hygroscopic then a layer of a waterproof covering is provided around it so that moisture can not reach the insulation. The main insulating materials which are in used are,

1. Poly Vinyl Chloride (PVC)
2. Paper
3. Cross Linked Polythene
4. Vulcanized India Rubber (VIR)

1.Poly Vinyl Chloride (PVC)

It is thermo plastic synthetic compound. It is available in the powder from and is obtained from polymerisation of acetylene. This powder is chemically inert, non-inflammable, odourless, tasteless and insoluble. It is combined with plastic compound and a gel is used over the conductor to obtain the insulation.

It has following characteristics,

1. Good dielectric strength of 17 kV/mm
2. Chemically inert.
3. Non-hygroscopic.
4. Resistant to corrosion.
5. Maximum continuous temperature rating of 75^{o}C.
6. High electrical resistivity.

The mechanical properties like elasticity of PVC are not as good as rubber so PVC cables are used for low and medium voltage domestic, industrial lights and power installations.

2.Paper

The paper is very cheap insulating material. Its dielectric strength is also high but it is hygroscopic in nature. When it is dry its insulation resistance is very high but a small amount of moisture reduces its insulation resistance to a very low value. Thus it is impregnated in an insulation oil. After impregnated also it has a tendency to absorb the moisture. Hence paper cables are never left unsealed and provided with the protective covering. When not in use, paper cable ends are temporarily covered with wax or tar.

The paper has following characteristics,

1. High dielectric strength of 20 kV/mm.
2. Higher thermal conductivity.
3. Low capacitance
4. High durability
5. Low cost
6. Maximum continuous temperature rating of 80^{o}C.
7. High insulation resistance when dry.

It is used in high voltage power cable manufacturing. The paper cables are preferred when the cable route has very few joints and hence generally used for low voltage distribution in thickly populated areas.

3. Cross Linked Polythelene

The cable using cross linked polythelene as the insulating material are called XLPE cables.

The low density polythelene is treated specially due to which there occurs cross linking of carbon atoms in it. This results into a new material which has following properties,

1. High dielectric strength of 20 to 40 kV/mm
2. Non-inflammable : If at all the continuous flame is applied its burning stops after very few centimeters away from the flame.
3. Extremely high melting point.
4. Light in weight and flexible.
5. Mechanically strong.
6. High temperature withstanding capability.
7. Low moisture absorption.
8. Maximum continuous temperature rating of 90^{o}C.

XLPE cables are directly laid on solid bed and are used for the voltage upto and including 33 kV.

4. Vulcanized India Rubber (VIR)

This is the most olden insulating material developed during 1880-1930. The pure rubber is very soft and it can not withstand high temperatures hence it is 20 to 40% of India rubber mixed with mineral matter such as zinc oxide, red lead etc. with a little bit of sulphur in it.

It has following characteristics,

1. Good dielectric strength.
2. Good mechanical strength.
3. Durable and wear resistant.
4. Good insulation resistance.
5. Remain more elastic than pure rubber.

But it has number of drawbacks such as,

1. It absorbs moisture, slightly.
2. It has low melting point.
3. The sulphur content attack the copper conductor and changes the VIR insulation colour. Hence copper conductors to be used with VIR insulation must be tinned.
4. Short span of life.

The use of VIR is very limited nowadays and is used for low moderate voltage cable i.e. distribution systems only.

Insulation Resistance of a Cable

The Fig 1 shows the section of a single core cable which is insulated with the help of layer of an insulating material.

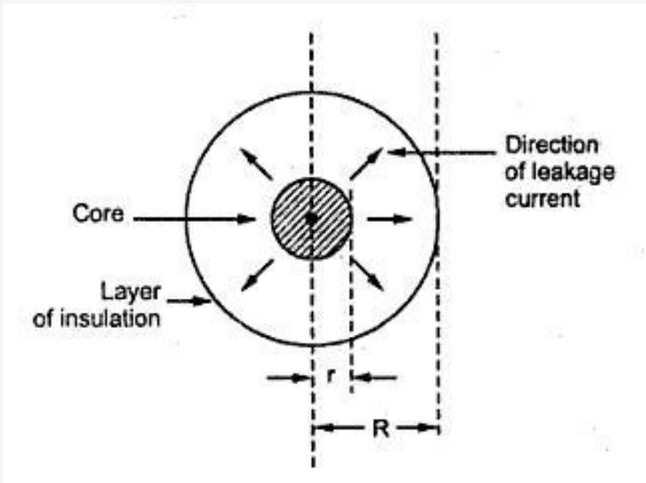


Fig. 1 Single core cable

In such cables, the leakage current flows radially from centre towards the surface as shown in the Fig.1. Hence the cross-section of the path of such current is not constant but changes with its length. The resistance offered by cable to path of the leakage current is called an insulation resistance consider an elementary section of the cylindrical cable of radius x and the thickness dx as shown in the Fig. 2. Let us find the resistance of this elementary ring.

Let d = Diameter of core

r = d/2 = Radius of core

D = Diameter with sheath

R = D/2 = Radius of cable with sheath

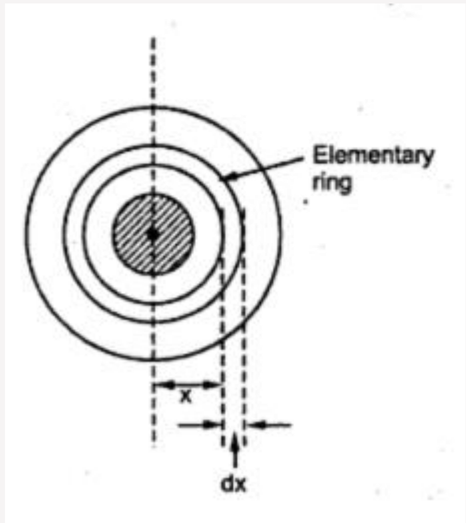


Fig. 2 Elementary ring

As the leakage current flows radially outwards, the length along which the current flows in an elementary ring is dx. While the cross-sectional area perpendicular to the flow of current depends on the length of l of the cable.

Cross-section area = Surface area for length l of cable

= (2 π x) x l

Hence the resistance of this elementary cylindrical shell is,

$$dR_i = \rho \cdot \frac{dx}{2\pi x l} \qquad \text{... As } R = \frac{\rho l}{A}$$

where ρ = Resistivity of the insulating material

The total insulation resistance of the cable can be obtained by integrating the resistance of an elementary ring from inner radius upto the outer radius i.e. r to R.

$$\therefore \qquad R_i = \int_r^R dR_i = \int_r^R \rho \frac{dx}{2\pi x l} = \frac{\rho}{2\pi l}\int_r^R \frac{dx}{x} = \frac{\rho}{2\pi l}[ln\, x]_r^R = \frac{\rho}{2\pi l}[ln\, R - ln\, r]$$

$$\therefore \qquad \boxed{R_i = \frac{\rho}{2\pi l}\, ln \frac{R}{r}\ \Omega}$$

This can be expressed interms of diameters as,

$$R_i = \frac{\rho}{2\pi l} \ln \frac{D}{d} \ \Omega$$

The value of R_i is always very high. The expression shows that the insulation resistance is inversely proportional to its length. So as the cable length increases, the insulation resistance decreases.

This shows that if two cables are joined in series then total length increases and hence their conductor resistances are in series giving higher resistance but insulation resistance are in parallel decreasing the effective insulation resistance. Thus if two cables are connected in parallel, conductor resistances get connected in parallel while the insulation resistance get connected in series.

Grading of Cables

We have seen that thew stress in the insulation is maximum at the conductor surface and minimum at the sheath. To avoid the breakdown of the insulation, it is necessary to have uniform distribution of stress all along the insulation.

Practically some methods are used to obtain uniform distribution of stress. The process of obtaining uniform distribution of stress in the insulation of cables is called grading of cables.

The unequal distribution of stress has two effects,

1. Greater insulation thickness is required, which increases the cost and size.
2. It may lead to the breakdown of insulation.

Hence the grading of cables is done.

There are two methods of grading the cables which are,

1. Use of intersheaths for grading
2. Capacitance grading

Let us discuss these two grading methods in detail

Use of Intersheath for Grading

In this method of grading, in between the core and the lead sheath number of metallic sheaths are placed which are called intersheaths. All these intersheaths are maintained at different potentials by connecting them to the tappings of the transformer secondary. These potentials are between the core potential and earth potential. Generally lead is used for these sheaths as it is flexible and corrosion resistance but as its mechanical strength is less, aluminium also can be used. Aluminium is low weight and mechanically strong but it is much costlier than lead.

Using the intersheaths, maintaining at different potential, uniform distribution of stress is obtained in the cables.

Consider a cable with core diameter d and overall diameter with lead sheath as D. Let two intersheaths are used having diameter d_1 and d_2 which are kept at the potentials V_1 and V_2 respectively.

The intersheaths and stress distribution is shown in the Fig. 1.

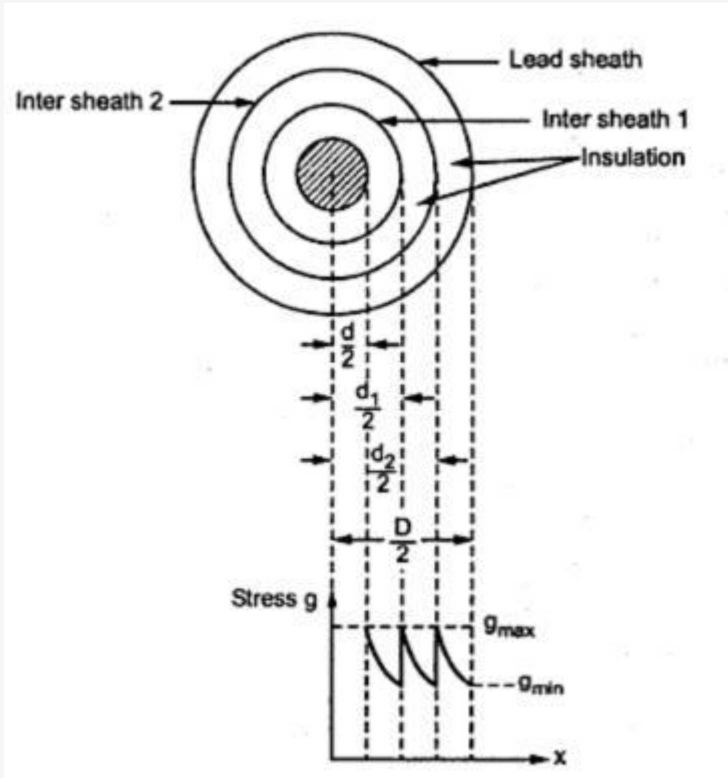


Fig. 1

Let V_1 = Voltage of intersheath 1 with respect to earth

V_2 = Voltage of intersheath 2 with respect to earth

It has been proved that stress at a point which is at a distance x is inversely proportional to distance x and given by,

$$g_x = \frac{Q}{2\pi\varepsilon x} = \frac{k}{x} \qquad \text{... (1)}$$

Where k is constant.

So electric stress between he conductor and intersheath 1 is,

$$g_1 = \frac{k_1}{x} \quad \text{where } k_1 = \text{Constant} \qquad \text{... (2)}$$

Now potential difference between core and the first intersheath is $V-V_1$.

$$\therefore \quad V - V_1 = \int_{d/2}^{d_1/2} g_1 \, dx = k_1 \int_{d/2}^{d_1/2} \frac{dx}{x} = k_1 [ln\, x]_{d/2}^{d_1/2} = k_1 \left[ln \frac{d_1}{2} - ln \frac{d}{2} \right]$$

$$= k_1 \, ln \left[\frac{d_1}{d} \right]$$

$$\therefore \quad k_1 = \frac{V - V_1}{ln\left(\frac{d_1}{d}\right)} \quad ...(3)$$

Substituting in equation (2) we get,

$$g_1 = \frac{V - V_1}{x\, ln\left(\frac{d_1}{d}\right)} \quad ...(4)$$

Now this stress is maximum at x = d/2, on core surface.

$$\therefore \quad \boxed{g_{1max} = \frac{V - V_1}{\frac{d}{2}\, ln\left(\frac{d_1}{d}\right)}} \quad ...(5)$$

Similarly potential difference between intersheath 1 and intersheath 2 is V_1 - V_2.

$$\therefore \quad g_2 = \frac{V_1 - V_2}{x\, ln\left(\frac{d_2}{d_1}\right)} \quad ...(6)$$

Now g_1 will be maximum at the surface of intersheath 1 i.e. x = $d_1/2$.

$$\therefore \quad \boxed{g_{2max} = \frac{V_1 - V_2}{\frac{d_1}{2}\, ln\left(\frac{d_2}{d_1}\right)}} \quad ...(7)$$

The potential difference between intersheath 2 and outermost sheath V_2 is only as potential of intersheath is maintained at V_2 with respect to earth.

$$\therefore \quad g_3 = \frac{V_2}{x\, ln\left(\frac{D}{d_2}\right)} \quad ...(8)$$

This g_3 will be maximum at x = $d_2/2$

$$\therefore \quad \boxed{g_{3max} = \frac{V_2}{\frac{d_2}{2}\, ln\left(\frac{D}{d_2}\right)}} \quad ...(9)$$

Choosing proper values of V_1 and V_2, g_{1max}, g_{2max} etc. can be made equal and hence uniform distribution of stress can be obtained.

The stress can be made to vary between same maximum and minimum values as shown in the Fig. 1, by choosing d_1 and d_2 such that,

$d_1/d = d_2/d_1 = D/d_2 =$

and $g_{1max} = g_{2max} = g_{3max}$

i.e. $$\frac{V-V_1}{\frac{d}{2}ln\alpha} = \frac{V_1-V_2}{\frac{d_1}{2}ln\alpha} = \frac{V_2}{\frac{d_2}{2}ln\alpha}$$

$$\therefore \quad \frac{V_2}{d_2\, ln\alpha} = \frac{V-V_2}{d_1\, ln\alpha} = \frac{V-V_1}{d\, ln\alpha}$$

$$\therefore \quad \frac{V_2}{d_2} = \frac{V-V_2}{d_1} = \frac{V-V_1}{d} \qquad \ldots (10)$$

$$\therefore \quad V_2 = \frac{d_2}{d_1}[V_1 - V_2] = \alpha\,[V_1 - V_2]$$

$$\therefore \quad V_2 = \alpha V_1 - \alpha V_2$$

$$\therefore \quad (1+\alpha) V_2 = \alpha V_1$$

$$\therefore \quad V_2 = \frac{\alpha}{1+\alpha} V_1 \qquad \ldots (11)$$

Let us try the express voltages V_1 and V_2 interms of V and .

Now $$\frac{V_1-V_2}{d_1} = \frac{V-V_1}{d}$$

$$\therefore \quad V_1 - V_2 = \frac{d_1}{d}(V - V_1) = \alpha\,(V - V_1)$$

$$\therefore \quad V_1 - V_2 = \alpha V - \alpha V_1$$

$$\therefore \quad (1+\alpha) V_1 = \alpha V + V_2 \qquad \ldots (12)$$

Substituting value of V_2 from equation (11),

$$(1+\alpha) V_1 = \alpha V + \frac{\alpha}{1+\alpha} V_1$$

$$\therefore \quad V_1 = \left[1+\alpha-\frac{\alpha}{1+\alpha}\right] = \alpha V$$

$$\therefore \quad V_1\left[\frac{(1+\alpha)(1+\alpha)-\alpha}{(1+\alpha)}\right] = \alpha V$$

$$\therefore \quad V_1\left[\frac{1+2\alpha+\alpha^2-\alpha}{1+\alpha}\right] = \alpha V$$

$$\therefore \quad V_1 = \frac{\alpha(1+\alpha)V}{\alpha^2+\alpha+1} = \frac{V\left(1+\frac{1}{\alpha}\right)}{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}} \qquad \ldots (13)$$

And $$V_2 = \frac{\alpha}{1+\alpha} V_1 = \frac{V_1}{\left(1+\frac{1}{\alpha}\right)} = \frac{V\left(1+\frac{1}{\alpha}\right)}{\left(1+\frac{1}{\alpha}\right)\left(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}\right)}$$

$$\therefore \quad V_2 = \frac{V}{\left(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}\right)} \qquad \ldots (14)$$

$$\therefore \quad g_{1max} = \frac{V-V_1}{\frac{d}{2} ln\left(\frac{d_1}{d}\right)}$$

$$\therefore \quad g_{1max} = \frac{V - \frac{V\left(1+\frac{1}{\alpha}\right)}{\left(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}\right)}}{\frac{d}{2} ln(\alpha)} = \frac{V\left[1 - \frac{1+\frac{1}{\alpha}}{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}}\right]}{\frac{d}{2} ln(\alpha)} = \frac{V\left[\frac{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}-1-\frac{1}{\alpha}}{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}}\right]}{\frac{d}{2} ln(\alpha)}$$

$$\therefore \quad g_{1max} = \frac{V}{\alpha^2\left(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}\right)\frac{d}{2} ln\alpha}$$

$$\therefore \quad g_{1max} = \frac{V}{(1+\alpha+\alpha^2)\left(\frac{d}{2}\right) ln\alpha} \qquad \ldots (15)$$

$$\frac{d_1}{d} \times \frac{d_2}{d_1} \times \frac{D}{d_2} = \alpha^3$$

$$\therefore \quad \frac{D}{d} = \alpha^3$$

$$\therefore \quad ln\left(\frac{D}{d}\right) = ln\ (\alpha^3) = 3\ ln\ \alpha$$

$$\therefore \quad ln\ (\alpha) = \frac{1}{3} ln\left(\frac{D}{d}\right)$$

$$\therefore \quad \boxed{g_{1max} = \frac{V}{\frac{1}{3}(1+\alpha+\alpha^2)\left(\frac{d}{2}\right) ln\left(\frac{D}{d}\right)}} \qquad \ldots (16)$$

Key Point : *If intersheath is not used,* g_{1max} *is* $\frac{V}{\frac{d}{2} ln\left(\frac{D}{d}\right)}$ *hence with intersheath it gets reduced by factor* $\frac{1}{\frac{1}{3}(1+\alpha+\alpha^2)}$.

Capacitance Grading

The grading done by using the layers of dielectrics having different permittivities between the core and the sheath is called capacitance grading.

In intersheath grading, the permittivity of dielectric is same everywhere and the dielectric is said to be homogeneous. But in case of capacitance grading, a composite dielectric is used.

Let d_1 = Diameter of the dielectric with permittivity ε_1

and D = Diameter of the dielectric with permittivity ε_2

This is shown in the Fig. 1.

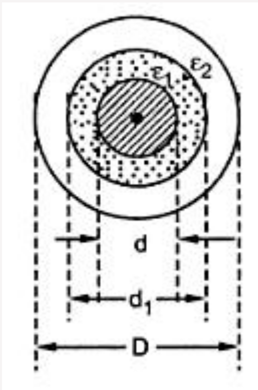


Fig. 1 Capacitance grading

The stress at a point which is at a distance x is inversely proportional to the distance x and given by,

$g_x = Q/(2\pi\varepsilon\, x)$

Hence the stress at point in the inner dielectric is,

$g_1 = Q/(2\pi\varepsilon_1\, x)$

Similarly the dielectric stress in the outer dielectric is,

$g_2 = Q/(2\pi\varepsilon_2\, x)$

Hence the total voltage V can be expressed as,

$$V = \int_{d/2}^{d_1/2} g_1\, dx + \int_{d_1/2}^{D/2} g_2\, dx = \int_{d/2}^{d_1/2} \frac{Q}{2\pi\varepsilon_1 x}\, dx + \int_{d_1/2}^{D/2} \frac{Q}{2\pi\varepsilon_2 x}\, dx$$

$$= \frac{Q}{2\pi}\left\{\frac{1}{\varepsilon_1}[\ln x]_{d/2}^{d_1/2} + \frac{1}{\varepsilon_2}[\ln x]_{d_1/2}^{D/2}\right\}$$

$$\therefore \quad V = \frac{Q}{2\pi}\left[\frac{1}{\varepsilon_1}\ln\frac{d_1}{d} + \frac{1}{\varepsilon_2}\ln\frac{D}{d_1}\right]$$

$$\therefore \quad C = \frac{Q}{V} = \frac{2\pi}{\frac{1}{\varepsilon_1}\ln\frac{d_1}{d} + \frac{1}{\varepsilon_2}\ln\frac{D}{d_1}}$$

The stress is maximum at surface of conductor i.e. x =d/2.

$$\therefore \quad g_{1max} = \frac{Q}{2\pi\varepsilon_1 \frac{d}{2}} = \frac{Q}{\pi\varepsilon_1 d}$$

And the stress is maximum at inner surface of dielectric i.e. at $x = d_1/2$.

$$\therefore \quad g_{2max} = \frac{Q}{2\pi\varepsilon_2 \frac{d_1}{2}} = \frac{Q}{\pi\varepsilon_2 d_1}$$

Substituting Q interms of V we get,

$$g_{1max} = \frac{2\pi V}{\pi\varepsilon_1 d\left[\frac{1}{\varepsilon_1} ln\frac{d_1}{d} + \frac{1}{\varepsilon_2} ln\frac{D}{d_1}\right]}$$

$$\therefore \quad g_{1max} = \frac{2V}{d\left[\frac{1}{\varepsilon_1} ln\frac{d_1}{d} + \frac{1}{\varepsilon_2} ln\frac{D}{d_1}\right]\varepsilon_1}$$

$$\therefore \quad g_{1max} = \frac{2V}{d\left[ln\frac{d_1}{d} + \frac{\varepsilon_1}{\varepsilon_2} ln\frac{D}{d_1}\right]}$$

$$\text{and} \quad g_{2max} = \frac{2V}{d_1\left[\frac{\varepsilon_2}{\varepsilon_1} ln\frac{d_1}{d} + ln\frac{D}{d_1}\right]}$$

Key Point : Thus the electric stress is inversely proportional to the permittivities and the inner radii of the dielectrics.

1.1 Condition for Equal Maximum Stress

Let us obtain the condition under which the maximum values of the stresses in the two regions are equal.

The maximum stresses are given by,

$g_{1max} = Q/(\pi\varepsilon_1 d)$

and $g_{2max} = Q/(\pi\varepsilon_2 d_1)$

Equating the two stresses,

$Q/(\pi\varepsilon_1 d) = Q/(\pi\varepsilon_2 d_1)$

$$\therefore \quad \boxed{\varepsilon_1 d = \varepsilon_2 d_1}$$

Now d_1 is greater than d so to satisfy above equation ε_2 must be less than ε_1.

Thus the dielectric nearest to the conductor must have the highest permittivity.

Similar for the grading with three dielectrics with permittivities ε_1, ε_2 and ε_3, for equal maximum stress the condition is,

And

$$\varepsilon_1 d = \varepsilon_2 d_1 = \varepsilon_3 d_2$$
$$\varepsilon_1 > \varepsilon_2 > \varepsilon_3$$

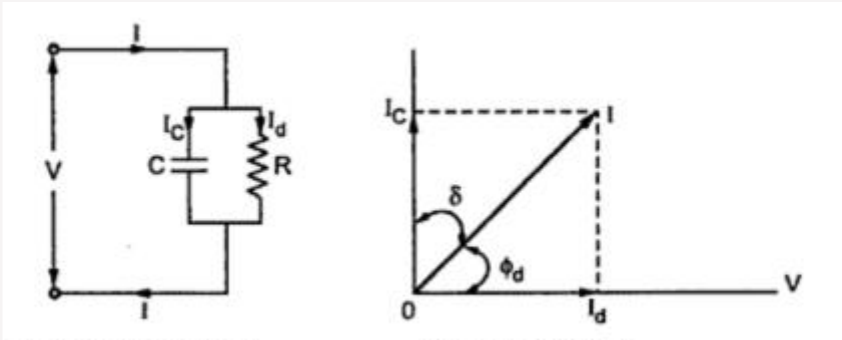
Difficulties in Grading

In the intersheath grading, the intersheath has to be thin. Hence there is possibility of damage to it while laying the cable. Similarly intersheath has to carry the charging current which can cause overheating of cable.

In capacitance grading, to have uniform distribution of stress it is necessary to select the dielectrics of proper permittivities. But practically it i difficult to get the proper values of permittivities. But practically it is difficult to get the proper values of permittivities. Similarly the permittivity of dielectric changes with the time which can cause uneven distribution of stress. Such uneven distribution may lead to breakdown at the normal operating voltage.

Dielectric Loss

There exists a capacitance between a conductor and the sheath, with a dielectric medium in between the two. This is represented as C. The leakage resistance is denoted as R. The equivalent circuit of the cable is a parallel combination of R and C. So there are two currents, one perpendicular to voltage V which is leading capacitive current I_c while other is in phase which voltage V which is resistive current I_d representing dielectric loss. This is shown in the Fig. 1(a) and (b).



(a) Equivalent circuit (b) Phasor diagram

Fig. 1

The dielectric loss is loss due to leakage resistance given by,

$$W = V^2/R$$

Now $\tan\delta = I_d/I_c = (V/R) / (V/X_c)$

$\therefore$ $V/R = (V/X_c) \tan\delta = V\omega C \tan\delta$

$$\boxed{W = V^2 \omega C \tan\delta}$$

Where δ = Dielectric loss angle in radius

Generally δ is very small and hence tan δδ. For low voltage cable dielectric loss can be neglected as it is small but for high voltage cables it must be considered.

The angle Φ_d is the power factor angle of dielectric.

$$\cos\Phi_d = \cos(90-\delta) = \sin\delta$$

It depends on the temperature and voltage stress to which dielectric is subjected.

Question Bank

1. What is corona? Derive expression for the disruptive critical voltage and visual critical voltage.
2. What are the effects of corona?
3. Write short note on disruptive critical voltage.
4. Discuss the advantages and disadvantages of corona.
5. Explain the terms with reference to corona.
   i) disruptive critical voltage ii) Power loss due to corona
6. Write short note on corona in transmission lines.
7. Write short note on factors affecting corona and methods of reducing corona effect.
8. Explain the terms with reference to corona.
9. Compare underground system with over head system.
10. A Single core cable is used on a 66KV, 3 Phase system. The core diameter is 1 cm while the insulation thickness is 1.5cm.If PVC of relative permittivity is 4.8 is used as dielectric.Calculate capacitance of cable and charging current.The supply frequency is 50 Hz.Assume cable length to be 1.5KM.
11. Briefly explain Murray loop test.

MODULE 5

MODULE-5

DISTRIBUTION

Requirements of a Good Distribution System

The necessary requirements of a good distribution system are,

1. The continuity in the power supply must be ensured. Thus system should be reliable.
2. The specified consumer voltage must not vary more than the prescribed limits. As per Indian Electricity Rules, the variation must not be beyond ± 5 % of the specified voltage.
3. The efficiency of the lines must be as high as possible.
4. The system should be safe from consumer point of view. There should no be leakage.
5. The lines should not be overloaded.
6. The layout should not affect the appearance of the site or locality.
7. The system should be economical.

Though the a.c. transmission and distribution is used, still for certain applications such as d.c. motors, electrochemical work, batteries, electric traction etc. the d.c. supply is must. Hence along with a.c., d.c. distribution is also equally important. In a d.c. distribution, d.c. generators are used in the generating stations or a.c. is converted to d.c. using the converters like mercury are rectifiers, rotary converters etc. at the substations. Then the d.c. supply is distributed to the consumers as per the requirement.

General D.C. Distribution System

The Fig. 1 shows a general distribution system in d.c. form where d.c. generators are used at the generating stations.

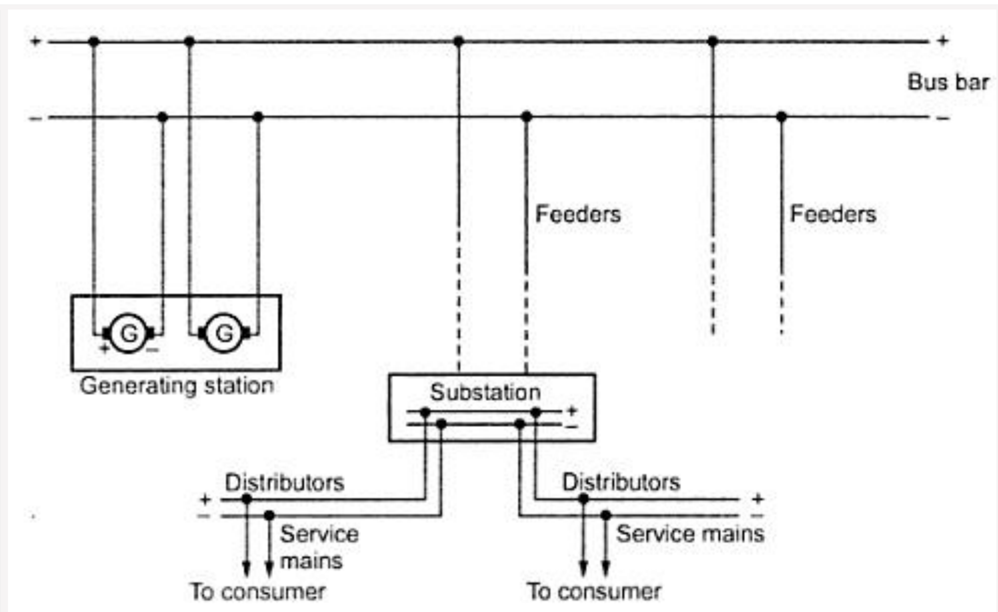


Fig. 1 General D.C. distribution system

As explained earlier, the feeders are used to feed the electrical power from the generating stations to the substations. The distributors are used to distribute the supply further from the substations. The service mains are connected to the distributors so as to make the supply available at the consumers permises. This is the simplest two wire distribution system used to supply the consumers. One more type of d.c. distribution system is also used in practice which is d.c. there wire system. Though for d.c. distribution, mainly two systems are used, the various types of distributors are used in these systems.

Radial Distribution System

The Fig. 1 shows a radial distribution system.

When the distributor is connected to substation on one end only with the help of feeder, then the system is called radial distribution system. The feeders, distributors and service mains are radiating away from the substation hence name given as radial system. There are combinations of one distributor and one feeder, connecting that distributor to the substation. In Fig. 1, distributor 1 is connected only at one end to substation through a feeder at point A. Similarly the other feeder is feeding the distributor 2, only at one point B.

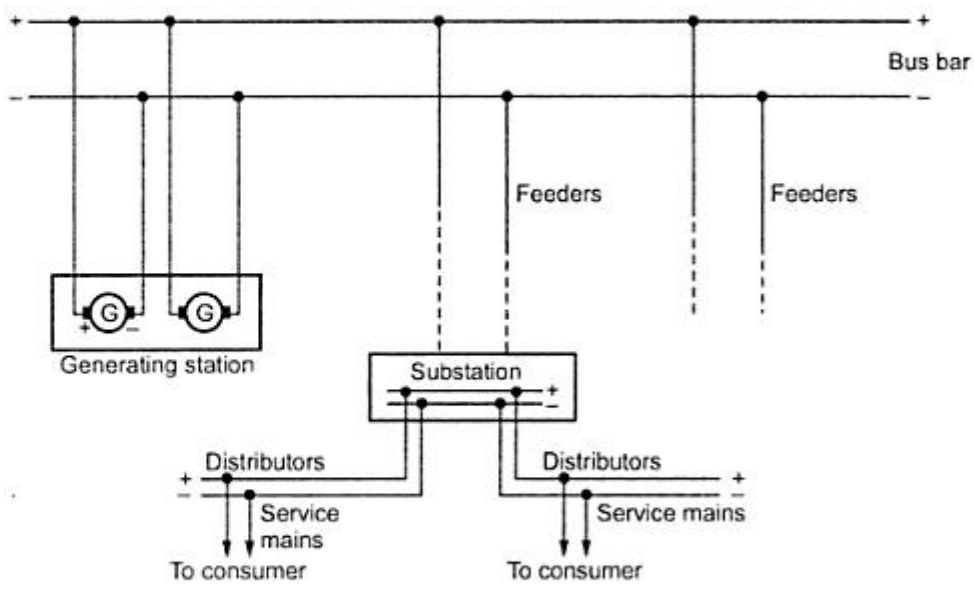


Fig. 1 Radial distribution system

Due to such system, if the fault occurs either on feeder or a distributor, all the consumers connected to that distributor will get affected. There would be an interruption of supply to all such consumers. Similarly the end of the distributor nearer to the substation will get heavily loaded than the end which is too far away from the substation. Similarly the consumers at the distant end of the distributor would be subjected to the voltage variations and fluctuations, as the load on the distributor changes. The system is advantageous only when the generation is at low voltage level and the substation is loaded at the center of the load.

The fault on a feeder or a distributor causes interruption in supply to all the consumers connected to the distributor. This can be avoided by modifying the radial system as shown in the Fig. 2. In this system, the distributor is fed at number of points with the help of feeders. In Fig. 2, the feeders from the substation are feeding to a single distributor at points A, B and C.

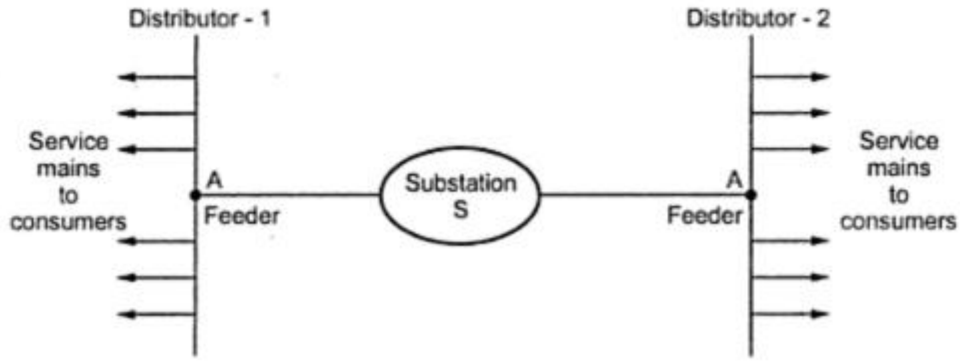


Fig. 2 Modified radial system

Advantages of Radial System

The various advantages of radial system are,

1. Simplest as fed at only end.
2. The initial cost is low.
3. Useful when the generating is at low voltage.
4. Preferred when the station is located at the centre of the load.

Disadvantages of Radial System

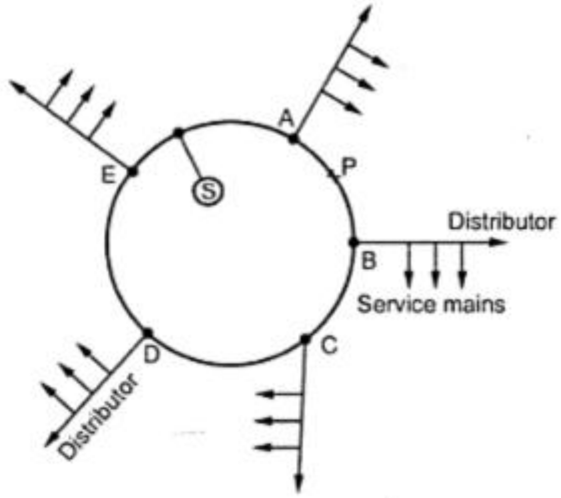
Apart from its advantages, this system is suffered from the following disadvantages.

1. The end of distributor near to the substation gets heavily loaded.
2. When load on the distributor changes, the consumers at the distant end of the distributor face serious voltage fluctuations.
3. As consumers are dependent on single feeder and distributor, a fault on any of these two causes interruption in supply to all the consumers connected to that distributor.

Ring Main Distribution System

Another system of distribution which eliminates the disadvantages of the radial system is used in practice called ring main distribution system.

In such system, the feeders covers the whole area of supply in the ring fashion and finally terminates at the substation from where it is started. The feeder is in closed loop from and looks like a ring hence the name given to the system as ring main system. This is shown in the Fig. 1.



The feeder in the ring fashion is divided into number of sections as AB, BC, CD, DE and EA. The various distributors are connected at A, B, C, D and E. Each distributor is supplied by the two feeders and hence the design is similar to the two feeders in parallel on different paths. Hence if there is any fault on any part of the feeder, still the consumers will keep on getting the continuous supply. For example, if the fault occurs at point P in the section AB of the feeder can be isolated and repaired. The feeder can be fed at one or more feeding points. Thus the disadvantages of radial system are eliminated in this system. The great saving in copper is another major advantage of the ring main system.

A.C. Distribution

In earlier days, d.c. system was used for the generation, transmission and distribution of electrical energy. But in case of d.c. system the voltage level cannot be changed easily unless we used rotating machinery which may not prove to be economical in many cases. This is the major disadvantages while working with d.c.

Later on with the development of transformer, a/c/ system has become predominant. Now days a large power systems in the world are using a.c. system rather than d.c. because of many advantages of a.c. system.

The transmission of electrical energy generated in the power station is at very high voltage with the use of 3 phase, 3 wire system. These voltages are stepped down for distribution at the substations. There are mainly two parts of the distributor system. They are primary distribution and secondary distribution. The voltage level of primary distribution system is higher than general utilization level. The secondary distribution systems receive power from primary distribution systems through distribution transformers. By distribution transformer voltage is stepped down to the normal working level and the consumers get the power with the voltage 400/230 V. The very commonly used a.c. distribution system is three phase four wire system as studied earlier.

A.C. Distribution Calculations

The A.C. distribution calculations and d.c. distribution calculations are different in the following respects :

1. In case of d.c. system, the voltage drop is due to resistance only which in a.c. system it is due to combined effect of resistance, inductance and capacitance.
2. The voltages or currents are added or subtracted arithmetically in case of d.c. system whereas they are added or subtracted vectorially in case of a.c. system.
3. It is required to take into account the power factor while making calculations in a.c. system which is absent in d.c. system. The distributors are normally tapped at different points with the loads having different power factors.

There are two ways of referring the power factor.

a) The p.f. may be referred to receiving end voltage which is reference vector.
b) The p.f. may be referred to the voltage at load point itself.

By different methods the a.c. distribution problems can be solved.

The most convenient method is the symbolic notation method wherein voltages, currents and impedances are expressed in the complex notation and the calculations are similar to those in case of d.c. distribution. In a.c. calculations, addition and subtraction must be done by expressing various quantities in the rectangular form while the multiplication and the division must be done by expressing the various quantities in the polar from.

Methods of Solving A.C. Distribution Problems

As discussed in earlier section of a.c. distribution system we have take into account the power factor. This power factor can be either considered with respect to receiving end voltage or with respect to load voltage itself. Let us consider each case separately.

1. Power Factors Referred to Receiving End Voltage

Consider an A.C. distribution PQ having concentrated loads of I_1 and I_2 tapped off at point Q and R respectively. This is shown in the Fig. 1.

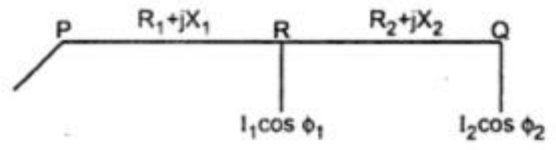


Fig. 1

Let voltage V_Q which is the voltage at the receiving end be taken as reference vector. The power factors at R and Q are $\cos\Phi_1$ and $\cos\Phi_2$ with respect to V_Q and they are lagging.

Let, R_1 = Resistance of section PR

X_1 = Reactance of section PR

R_2 = Resistance of section RQ

X_2 = Reactance of section RQ

Impedance of section PR is given by,

$$\bar{Z}_{PR} = R_1 + j X_1$$

Impedance of section RQ is given by,

$$\bar{Z}_{RQ} = R_2 + j X_2$$

The load current at point R is $\bar{I}_1$,

$$\bar{I}_1 = I_1 \angle -\phi_1 = I_1 (\cos \phi_1 - j \sin \phi_1)$$

Similarly the load current at point Q is $\bar{I}_2$,

$$\bar{I}_2 = I_2 \angle -\phi_2 = I_2 (\cos \phi_2 - j \sin \phi_2)$$

The current in section RQ is nothing but $\bar{I}_2$,

$$\therefore \quad \bar{I}_{RQ} = \bar{I}_2 = I_2 (\cos \phi_2 - j \sin \phi_2)$$

The current in section PR is given by,

$$\bar{I}_{PR} = \bar{I}_1 + \bar{I}_2$$

$$= I_1 (\cos \phi_1 - j \sin \phi_1) + I_2 (\cos \phi_2 - j \sin \phi_2)$$

The voltage drop in section RQ is given by,

$$\bar{V}_{RQ} = \bar{I}_{RQ} \bar{Z}_{RQ}$$

$$\therefore \quad \bar{V}_{RQ} = [I_2 (\cos \phi_2 - j \sin \phi_2)] \cdot [R_2 + j X_2]$$

The voltage drop in section PR is given by,

$$\bar{V}_{PR} = \bar{I}_{PR} \bar{Z}_{PR}$$

$$= [I_1 (\cos \phi_1 - j \sin \phi_1) + I_2 (\cos \phi_2 - j \sin \phi_2)] [R_1 + j X_1]$$

Thus the sending end voltage V_A is given as,

$$\bar{V}_P = \bar{V}_Q + \bar{V}_{RQ} + \bar{V}_{PQ}$$

The sending end current is given as,

$$\bar{I}_P = \bar{I}_1 + \bar{I}_2$$

The corresponding phasor diagram is shown in the Fig. 2.

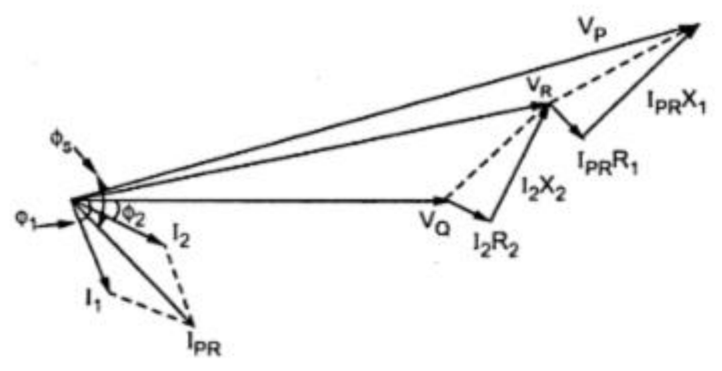


Fig. 2

As shown in the Fig. 2. the receiving end voltage V_Q is taken as reference vector. The currents I_1 and I_2 are lagging from V_Q by angles ofΦ_1 and Φ_2 respectively. The vector sum of I_1 and I_2 gives current I_{PR}. The drop is I_2R_2 in phase with I_2 while I_2X_2 is leading by 90^o. The vector sum of V_Q, I_2R_2 and I_2X_2 gives V_R. The drop $I_{PR}R_1$ is in phase with current I_{PR} while $I_{PR}X_1$ is leading by 90. The vector sum of V_R, $I_{PR}R_1$ and $I_{PR}X_1$ gives the sending end voltage.

2. Power Factors Referred to Respective Load Voltages

In previous section we have considered the load power factors with respect to receiving end voltage. Here we will consider these power factors with respect to their respective load voltages. Now Φ_1 is the phase angle between V_R and I_1 while the angle Φ_2 is the phase angle between V_Q and I_2.

The phasor diagram under this condition will be as shown in the Fig. 3.

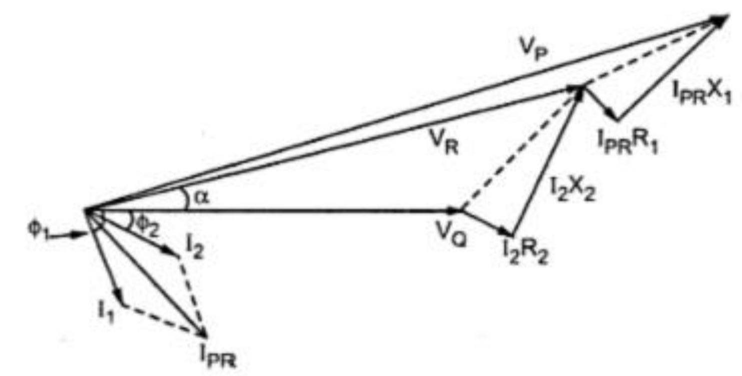


Fig. 3

Here again the receiving end voltage V_Q is the reference phasor. The vector sum of I_1 and I_2 gives the current I_{PR}. The drop I_2R_2 is in phase with I_2 while I_2X_2 is leading by 90. The vector sum of V_Q, I_2R_2 and I_2X_2 gives voltage V_R. The drop $I_{PR}R_1$ is in phase with current I_{PR} while the drop $I_{PR}X_1$ is leading by 90^o. The vector sum of V_R, $I_{PR}R_1$ and $I_{PR}X_1$ gives the sending end voltage V_p.

Now voltage drop in section RQ is given by, Voltage drop in section RQ $= \bar{I}_2 \bar{Z}_{RQ}$

$\therefore \quad \bar{V}_{RQ} = [I_2 (\cos \phi_2 - j \sin \phi_2)] [R_2 + j X_2]$

Voltage at point R, $\bar{V}_R = V_Q +$ Drop of voltage in section RQ

$= V_R \angle \alpha$ (say)

We have, $\bar{I}_1 = I_1 \angle -\phi_1$ w.r.t. voltage V_R

$\bar{I}_1 = I_1 \angle -(\phi_1 - \alpha)$ w.r.t. voltage V_Q

$\therefore \quad \bar{I}_1 = I_1 [\cos (\phi_1 - \alpha) - j \sin (\phi_1 - \alpha)]$

Now, $\bar{I}_{PR} = \bar{I}_1 + \bar{I}_2$

$= I_1 [\cos (\phi_1 - \alpha) - j \sin (\phi_1 - \alpha)] + I_2 [\cos \phi_2 - j \sin \phi_2]$

Voltage drop in section PR $= \bar{I}_{PR} \bar{Z}_{PR}$

$$\therefore \quad \bar{V}_{PR} = \{I_1 [\cos (\phi_1 - \alpha) - j \sin (\phi_1 - \alpha)] + I_2 [\cos \phi_2 - j \sin \phi_2]\} \cdot [R_1 + j X_1]$$

$\therefore$ The sending end voltage V_p is given by,

$\bar{V}_P = \bar{V}_Q$ + voltage drop in RQ + voltage drop in PR

$$\therefore \quad \bar{V}_P = \bar{V}_Q + \bar{V}_{RQ} + \bar{V}_{PR}$$

Question Bank

1. Write short note on radial and ring main distributors.
2. What is meant by DC distribution? Explain with diagram different types of DC distribution and discuss their merits and demerits.
3. Write short note on radial distribution system.
4. Show different types of distribution systems with single line diagrams and state the merits and demerits of ring main and radial distribn. Systems.
5. Calculate the total voltage drop in uniformly loaded distributor, when it is fed at one end.
6. Write short note on ring main distributors.